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COVER SHEET FOR TECHNICAL MEMORANDUM

TITLE- Determination of the Position of an Extravehicular Crewman With Respect to the LM on the Lunar Surface

TM- 69-2034-9

DATE- October 23,1969

FILING CASE NO(S)- 320

AUTHOR(S)- A. G. Weygand

FILING SUBJECT(S)-
(ASSIGNED BY AUTHOR(S)- Relative Position Determination

ABSTRACT

Methods for the determination of the relative position of an extravehicular crewman (EVC) with respect to the Lunar Module (LM) on the lunar surface using Earth-based or lunar-based tracking systems are examined and compared. The types of lunar-based tracking systems examined included line-of-sight relay tracking systems and over-the-horizon ground wave tracking systems. Four different types of baseline tracking methods assuming the use of the Unified S-Band (USB) stations of the Manned Space Flight Network (MSFN) located at Ascension, Bermuda and Goldstone are discussed. Determination of the relative position of the LM and EVC on the lunar surface with an uncertainty of approximately 300 meters is possible through use of these stations if the random uncertainty in the slant range measurements made from these stations is no greater than 1 meter. However, the actual bias and random error statistics on slant range measurements made by USB stations of the MSFN must be determined before a truly valid estimation of the uncertainty in relative position determination provided through use of these methods can be made.

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(NASA-CR-109072) DETERMINATION OF THE POSITION OF AN EXTRAVEHICULAR CREWMAN WITH RESPECT TO THE LM ON THE LUNAR SURFACE (Bellcomm, Inc.) 57 p

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DATE: October 23, 1969

FROM: A. G. Weygand

TECHNICAL MEMORANDUM

1.0 INTRODUCTION

In order to enhance the scientific yield from the missions of the Lunar Exploration Program (LEP), knowledge of the precise location on the surface of the Moon where soil and/or rock samples were obtained or where lunar surface experiments were deployed is required. Although use of map matching techniques will almost certainly be necessary to obtain the desired location accuracy, a method for position determination must be provided with sufficient accuracy to make the use of map matching techniques feasible. Aside from this requirement is the crew safety requirement for continuous knowledge of the relative position of one or two crewmen performing extravehicular activity (EVA) on the lunar surface and the Lunar Module (LM) with sufficient accuracy to permit the crewmen to return directly to within line-of-sight of the LM without performing a search or to enable the rescue of one crewman by another. In the proposed missions of the LEP, traverses by the crewmen from the LM extending out to a radius of 5 kilometers are being considered.

A number of relative position determination methods have been suggested for locating the extravehicular crewmen (EVC) with respect to the LM. Some of the suggested methods, which are listed below, rely on the assumption that the precise location of the LM on the lunar surface can be determined by independent means and is known.

(a) LM-located scanning television system.

(b) Crewman-mounted inertial guidance and navigation system.

(c) Earth-based electronic tracking systems.

(d) Lunar-based electronic and laser tracking systems.

These methods for relative position determination are discussed in the following paragraphs of section 2.0. A summary is provided in section 3.0.

2.0 DISCUSSION

Further consideration of using a scanning television system at the LM for relative position determination of an EVC was abandoned because results of a study by Rosenblum[(1)] indicate that a significant amount of line-of-sight masking will exist between a LM with a 500 foot mast and an EVC equipped with a 50 foot mast for arbitrary traverses by a crewman within a circle of 5 kilometers radius from the LM at a typical scientific lunar landing site. In addition, the resolution provided by the television camera of the type which will be included in the Lunar Surveying System assuming the use of a telephoto lens providing a 9 degree field of view will not be sufficient to distinguish the EVC from the lunar surface background or from electrical noise in the television picture when the crewman is within line-of-sight of the LM but 5 kilometers distant.

The potential accuracy of relative position determination provided by specific tracking systems and techniques listed above are discussed in this memorandum. Discussion of the potential accuracy of relative position determination provided by a crewman-mounted inertial guidance and navigation system is beyond the scope of this memorandum. However, a cursory look at this relative position determination technique indicates that a more detailed investigation is warranted.[(2)] Discussion of celestial navigation and landmark navigation is also beyond the scope of this memorandum.

The general approach taken to arrive at the uncertainty in the relative position determination made by a specific tracking system was first to derive an explicit set of equations defining the relative location of the EVC with respect to the LM in terms of known and/or measurable parameters. It was assumed that all parameters in these derived equations were mutually independent. Thus, assuming X_s (the X-axis coordinate of the position of the EVC with respect to the LM) is a function of the variables X_1 and R_1 which are independent, the true increment or uncertainty in X_s resulting from a change or measurement uncertainty in X_1 which is defined as

$$\Delta X_s = f(X_1+\Delta X_1, R_1) - f(X_1, R_1)$$

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(1) I. I. Rosenblum, "Screening of Line of Sight to LM by Craters at Apollo Site 2-Mission G," Memorandum for File, June 30, 1969.

(2) Private communication with Mr. R. V. Sperry, Bellcomm, Inc.

is given by

$$\Delta X_s = \frac{\partial X_s}{\partial X_1} \Delta X_1 = \frac{\partial f(X_1, R_1)}{\partial X_1} \Delta X_1$$

and the uncertainty in X_s as a result of a measurement uncertainty in R_1 is given by

$$\Delta X_s = \frac{\partial X_s}{\partial R_1} \Delta R_1 = \frac{\partial f(X_1, R_1)}{\partial R_1} \Delta R_1$$

and the total uncertainty in X_s as a result of measurement uncertainties in both X_1 and R_1 is given by

$$\Delta X_s = \left[\left(\frac{\partial X_s}{\partial X_1} \Delta X_1 \right)^2 + \left(\frac{\partial X_s}{\partial R_1} \Delta R_1 \right)^2 \right]^{1/2}$$

Equations are given in Appendix A which define the position of a point in an arbitrary rectangular coordinate system and the uncertainty in this position determination in terms of known or measurable quantities. It should be noted that the error equations contained in Appendix A give the uncertainty in the three coordinates of the determined position of the point in the arbitrary rectangular coordinate system chosen. The uncertainties in the location of the origin of this arbitrary rectangular coordinate system with respect to a fixed reference coordinate system have not been addressed because for the purposes of this memorandum these equations will be used for the determination of the relative position of two points rather than their absolute position. Equations are given in Appendix B which define the relative position of a point with respect to a second point in an arbitrary rectangular coordinate system and the uncertainty in the determination of this relative position in terms of known or measurable quantities.

The equations contained in Appendices A and B are used in the following paragraphs to assess the potential accuracy of the determination of the relative position of an EVC with respect to the LM using parameter values and accuracies provided by various Earth-based electronic tracking system configurations and various lunar-based electronic and laser tracking systems.

2.1 Earth-Based Electronic Tracking Systems

The uncertainty in the determination of the relative position of an EVC with respect to the LM was examined using the following four sets of data, each one of which could be provided by properly configured Earth-based tracking systems:

(a) two sets (one set for the crewman and the second set for the LM) of three slant range measurements.

(b) two sets of three slant range sum measurements.

(c) three difference measurements of slant range sums.

(d) two sets of one slant range and two slant range difference measurements.

The uncertainty in the relative position determination provided by these combinations of data was calculated for only one position of the Moon with respect to the Earth and one position of the EVC with respect to the LM (assumed to be a 5 kilometer separation) on the lunar surface using data provided by stations of the Manned Space Flight Network (MSFN) located at Goldstone, Bermuda and Ascension. The relative position of the Moon was chosen so that the LM would be equidistant from the three MSFN stations. The uncertainty in the location of the MSFN stations in each of the three axes of a geocentric rectangular coordinate system is approximately 35 meters for the Goldstone station, 40 meters for the Bermuda station, and 105 meters for the Ascension station.[3] For all cases, Bermuda was assumed to be the central station. For the above cases (a), (b), and (d), the origin of the reference rectangular coordinate system will be placed at the Bermuda station, the X-axis will pass through the Ascension station, and the Goldstone station will lie in the XY-plane. For case (c), the origin of the arbitrary reference rectangular coordinate system will be placed at the LM location and the Z-axis will pass through the Bermuda station.

Items peculiar to each one of these four cases including a brief treatment of the uncertainties of the tracking system measurements are discussed in the following paragraphs. Obviously for all four cases, line-of-sight between the MSFN stations and the LM and between the stations and the EVC is required.

2.1.1 Three Slant Range Measurements

For this case, measurements of slant range between an MSFN station and the LM and between that station and the EVC

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(3) GSFC, X-832-69-69, "MSFN Metric Tracking Performance Report AS-503," February, 1969.

are required simultaneously from at least three MSFN stations which are as widely spaced as possible. The required slant range data could be obtained through use of the pseudo noise ranging capability of the Unified S-Band (USB) system or by integration of the Doppler frequency shift counts provided through use of the slant range rate measuring capability of the USB system. Consequently, the LM and the EVC would each be required to carry three S-band transponders with each of the six transponders being tuned to a different set of received and transmitted carrier frequencies in order to avoid RF interference. It should be noted that requirements for these transmit and receive frequency capabilities at the stations of the MSFN would require additional RF bandwidth to be allocated for space use by the Director of Telecommunications Management (DTM) and would require some new transmitters, receivers, and antenna feeds to be added to the appropriate stations to accommodate the larger number of links and the different carrier frequencies. If the EVC and LM were each equipped with only one transponder, the necessary slant range data could be obtained by the MSFN stations using their pseudo noise ranging capability and time sharing the EVC and LM transponders. However, the uncertainty in the determination of relative position would be increased over that obtained using simultaneous measurements because the EVC could have changed position significantly before slant range measurements have been made by the three MSFN stations (estimated to be of the order of 5 minutes including link acquisition and integration times for the signal to noise ratios used in the Apollo Program).

In general in the USB system, slant range from an MSFN station to a transponder is determined by transmitting a digital sequence (called a pseudo noise code) at a clock rate of 1 MHz from the MSFN station to the transponder where the sequence is turned-around and transmitted back to the originating station. The received sequence is compared to a record of the transmitted sequence and the delay of the sequence introduced by the round-trip between an MSFN station and a transponder is determined. This delay corrected for fixed delays in equipments such as the transmitter, transponder, receiver, wiring, etc., is proportional to the slant range sum from an MSFN station to a transponder and back to the MSFN station.

In the USB system, two-way slant range rate of a transponder with respect to an MSFN station is determined by transmitting a continuous wave carrier from the MSFN station to the transponder where the received carrier frequency is translated in phase coherence through a constant ratio to a different frequency and transmitted back to the originating station. This received carrier frequency is compared with a sample of the USB transmitter reference frequency which has also been translated in phase coherence through the same constant ratio used in the transponder. The difference in frequency of these two carriers (or Doppler frequency shift) is proportional to the two-way slant range rate of the transponder with respect to the MSFN station.

To use data provided by integration of the Doppler frequency shift counts, the data must be initialized using known positions of both the LM and EVC at some given time as references. In addition, this data must be reinitialized whenever the count of the number of Doppler cycles is lost or interrupted such as would occur if frequency lock was lost in either the MSFN station receiver or transponder.

Uncertainties in time synchronization (up to $\pm$ 8 millisec [3]) and frequency standard synchronization (up to $\mp$ 5×10^{-12} Hz [3]) among the various stations of the MSFN as well as uncertainties in the velocity of electromagnetic wave propagation in space and atmosphere will most likely introduce significant bias errors in the determination of the absolute position of either the LM or the EVC. However, these uncertainties will have negligible effect on the determination of their relative position because slant range measurements to the LM and to the EVC will be made simultaneously by an MSFN station and because the slant ranges from the MSFN stations to the LM will be very much greater than the separation of the EVC and LM. Therefore, these contributors of measurement bias errors will not be considered further in this section.

During the translunar coast phase of the Apollo-Saturn mission AS-503, the root-mean-square random error for slant range measurements from stations of the MSFN to the Command and Service Module (CSM) was found to be approximately 15 meters.[3] Also important to the estimation of the accuracy of the relative position determination when using the pseudo noise code to determine range is the potential difference in delay fluctuations between the S-band transponders carried by the LM and the EVC caused by among other things differences in the stability of the thermal environment of the transponders as the crewman moves away from the LM. The specification [4] for the time delay through the ranging channel of a USB transponder for the Apollo CSM states that "the time delay through the transponder shall not exceed 2000 nanoseconds, and shall vary no more than 100

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[4] Collins Radio Company, "Equipment Specification for the Unified S-Band Equipment of the Apollo Communications and Data Subsystem," 514-0007-001, Revision D, October 14, 1965.

nanoseconds under any combination of environments and radio frequency input levels from -50dBm to -120dBm and for Doppler frequency offset of ±90 kHz." The corresponding specification[5] for the time delay of the pseudo noise ranging signal through the transponder, diplexer and power amplifier of the Apollo LM is that the time delay shall not exceed 2.1 microseconds and shall have a maximum variation of 100 nanoseconds. It should be noted that a delay uncertainty of 50 nanoseconds corresponds to a slant range measurement uncertainty of 7.5 meters or to a slant range sum measurement uncertainty of 15 meters. For the purposes of this section, it was assumed that the uncertainty in a slant range measurement between a station of the MSFN and a transponder carried by the LM or a crewman provided by the current USB system using the pseudo noise code was 15 meters.

The precise statistics of the random error in the Doppler frequency shift measurements and subsequent integration to yield a measurement of change in slant range from the reference station were not known to the writer, although it was reported[3] that the standard deviation of the range rate data taken during translunar and transearth mission phases of AS-503 was consistently less than 3 millimeters per second. Since, in general, the bias errors present in the measurements of slant range using this method from an MSFN station to the LM and to the EVC will be approximately the same, their contribution to the uncertainty in the determination of relative position of the EVC and LM should be negligible. For the purposes of this memorandum, it was assumed that the standard deviation of the random error in slant range measurements through integration of Doppler frequency shift counts for relatively long periods of time was 1 meter. It is recognized that further investigation of the uncertainties in this type of slant range measurement, including the effects of movements by the EVC other than translational, is required before its use could be adopted.

The predicted uncertainty in the determination of the relative position of an EVC with respect to the LM on the lunar surface using two sets of three slant ranges was calculated using the error equations in section 1.1 of Appendix A and section 1.0 of Appendix B under the conditions described above. The results of these calculations appear in Table 1.

2.1.2 Three Slant Range Sum Measurements

For this case, simultaneous measurements of the slant range sum from one station of the MSFN to the LM and back to any station of the MSFN for at least three combinations of MSFN

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[5] Grumman Aircraft Engineering Company, "Communications Subsystem Design Control Specification For," LSP-380-2B, May 20, 1966.

stations are required simultaneously with corresponding measurements of the slant range sum to an EVC using the same MSFN stations. The required slant range sum data could be obtained through use of the pseudo noise ranging capability of the USB system or by integration of the Doppler frequency shift counts provided through use of the slant range rate measuring capability of the USB system as discussed in the previous section. RF transmissions would be made from one MSFN station simultaneously to both the LM and the EVC. The S-band signal received at the LM and that received at the EVC would each be turned-around and retransmitted toward the Earth at different carrier frequencies, each coherently related to the respective received carrier frequency. These retransmitted S-band signals would be received by at least three stations of the MSFN including the station which was used to transmit the original S-band signals. Therefore, the LM and the EVC would each be required to carry only one S-band transponder.

If the pseudo-noise ranging capability of the USB system were used to determine slant range sum data, digital sequences identical to those transmitted to the LM and the EVC would have to be generated locally at all stations other than the originating station. Clock frequencies derived from the pseudo-noise codes received from the LM and the EVC must be used to drive the digital sequence generators at those stations. The locally generated digital sequences will be compared with the received sequences and relative delays between the reference and received sequences will be measured. In order for the measured delays to be proportional to the true slant range sum, the digital sequences generated at all stations must use a common reference starting point. This reference could be established by making slant range sum measurements when the precise location of a transponder is known and adjusting the sequence start reference at each station so that the proper value of the slant range sum measurement is achieved.

If the slant range rate measuring capability of the USB system were used to determine slant range sum data by integration, the frequency standards at all stations of the MSFN must be identical and precisely known so that the measured Doppler frequency counts are a true representation of the slant range rate sum. If the primary frequency standard at the receiving site is offset with respect to the standard at the transmitting site, the resulting difference in cycles accumulated over the tally period will be added algebraically to the Doppler frequency counts producing a bias error. To use slant range sum data provided by integration of Doppler frequency counts, the data must be initialized using known positions of both the LM and an extravehicular crewman at some given point in time as a reference starting point.

Although uncertainties in time synchronization and frequency standard synchronization among the various stations of the MSFN will introduce bias errors in the determination of the absolute position of either the LM or the extravehicular crewman, they will have negligible effect on the determination of their relative position.

As indicated in the previous section, delay fluctuations in the turn-around channel of the LM S-band transponder which are different than those in the turn-around channel of the S-band transponder of the EVC will introduce errors in their relative position determination. However, it was assumed that any differential change in delay through the two transponders would be included in the random uncertainty in a slant range sum measurement to the LM or to the EVC obtained using the pseudo noise code which was assumed to be 15 meters.

Ignoring the bias error effects of frequency offset of the primary frequency standards of different MSFN stations and scale factor uncertainties since they would be the same for slant range sum measurements to the LM and to the EVC, it was assumed that the standard deviation of the random error in slant range sum measurements provided by integration of Doppler frequency shift counts for relatively long periods of time was 1 meter.

The predicted uncertainty in the determination of the relative position of an EVC with respect to the LM on the lunar surface using two sets of three slant range sum measurements was calculated using the error equations in Section 1.2 of Appendix A and Section 1.0 of Appendix B under the conditions described above. The results of these calculations appear in Table 1.

2.1.3 Three Difference Measurements of Slant Range Sums

For this case, simultaneous measurements of the difference between the slant range sum from one station of the MSFN to the LM and back to any station of the MSFN and the corresponding slant range sum to an EVC using the same MSFN stations are required from at least three combinations of MSFN stations. This case which was suggested by James[6] is an extension of the case discussed in Section 2.1.2 where two sets of three slant range sums were used for relative position determination.

RF transmissions would be made from one MSFN station simultaneously to both the LM and the EVC. In the configuration suggested by James, the same digital sequence would be transmitted in synchronization to both the LM and the EVC. The sequence received at the LM and that received at the EVC would each be turned-around and be transmitted to the Earth on carriers of different frequencies to avoid RF interference. These two S-band signals would be received by at least three stations of the MSFN including the station which transmitted the digital sequences originally. Consequently, the LM and the EVC would each be required to carry only one S-band transponder.

To measure directly the difference between the slant range sums to the LM and to the EVC, the applicable stations of the MSFN would each have to be modified to permit a digital sequence detected on one S-band channel used by the LM to be compared with digital sequence detected on a second S-band channel used by the EVC and to determine the differential delay in the two sequences. This differential delay corrected for various known fixed delays in the transmission and reception circuitry will be proportional to the difference in the slant range sums to the LM and to the EVC from two MSFN stations.

As was pointed out in the previous two sections, differential changes in delay through the LM S-band transponder with respect to the S-transponder of the EVC will influence the accuracy of the relative position determination of the two transponders while uncertainties in station time synchronization and in wave propagation velocity will not. It was assumed that the uncertainty in measuring the difference between two range sums as described above was 21 meters. This figure includes the uncertainty due to any differential delay in the two transponders.

Alternatively, the difference between the slant range sum to the EVC and the slant range sum to the LM could be obtained by measuring the slant range sums as described in Section 2.1.2 and subtracting them at each station. The uncertainty

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[6] D. B. James, "A Method of Navigating On and Near the Moon," Memorandum for File, June 28, 1968.

in determining the difference between two range sums in this manner would be 21 meters if the pseudo noise ranging capability of the MSFN USB stations were used and 1.4 meters if integration of Doppler frequency shift counts were used to make the individual slant range sum measurements.

The predicted uncertainty in the determination of the relative position of an EVC with respect to the LM on the lunar surface using three difference measurements of slant range sums was calculated using the error equations in Section 2.0 of Appendix B under the conditions described above. The results of these calculations appear in Table 1.

2.1.4 One Slant Range Measurement and Two Difference Measurements of Slant Ranges

For this case, simultaneous measurements of slant range between one MSFN station and the LM and of the difference between the slant range from the LM to that station and the slant range from the LM to each of two other stations must be made simultaneously with corresponding measurements using the same MSFN stations with the EVC. As indicated earlier in section 2.1.1, the required slant range data could be obtained through the use of the ranging capability of the USB system or by integration of the Doppler frequency shift counts provided through use of the slant range rate measuring capability of the USB system.

To measure the differences in slant ranges, it is proposed that a digital sequence be transmitted from the LM on one carrier frequency and a similar digital sequence be transmitted from the EVC on a second carrier frequency. Two digital sequences must be generated at the MSFN stations identical to those transmitted from the LM and the EVC. The clock frequencies required to drive the two digital sequence generators must be derived from the two digital sequences received by the MSFN station. The "on-station" generated digital sequences would then be compared with the corresponding sequences received from the LM and the EVC. The result of these comparisons will be two arbitrary delay measurements at each station. A reference point for all future measurements must be obtained at some time when the relative position of the LM and the EVC is known. After initialization, the clocks of the digital sequence generators at the LM and the EVC must be stable or have negligible relative drift and the delay variation in the equipments carried by the LM and the EVC as well as in the MSFN stations used in the LM and the EVC channels must be negligible. Only then will changes in the slant range difference between the LM or the EVC and two MSFN stations be proportional to the relative change between the appropriate delay measurements at each of the two stations. The LM and the EVC would each be required to carry one S-band transponder, one digital sequence or pseudo noise code generator, and an RF transmitter for transmitting the digital sequence.

The clock rate of the digital sequences transmitted by the LM and the EVC must be at least an order of magnitude greater than the clock rate used by the existing pseudo noise code generators of the USB stations (1 MHz) in order to obtain the necessary resolution and accuracy of the slant range difference determination. Hence, two new wideband RF channels (at least 10 to 20 MHz bandwidth for each) would have to be allocated for space use by the DTM and appropriate stations of the MSFN would have to be modified to accommodate these channels and to process the detected digital sequences as described earlier to obtain measures of the differences of slant ranges.

Although uncertainties in time synchronization among the various MSFN stations and in the synchronization of the clocks of the digital sequence generators of the MSFN stations, the LM, and the EVC will introduce bias errors in the determination of the absolute position of either the LM or the EVC, they will have negligible effect on the determination of their relative position so long as these uncertainties remain constant after initialization. Any differential change in delay in the circuitry of the two channels occurring after initialization would introduce bias erros in the relative position determination.

As was done in the previous sections, the random uncertainty in a slant range measurement provided by the current USB system using the pseudo noise code ranging capability was assumed to be 15 meters. It was assumed that the random uncertainty in the determination of the difference between two slant ranges including the effects of changes in differential delay in the two channels after initialization due to circuitry delay drifts and clock synchronization drift would be in the range of 0.1 meter to 1.0 meter.

The predicted uncertainty in the determination of the relative position of the EVC with respect to the LM on the lunar surface using two sets of one slant range and two slant range difference measurements was calculated using the error equations in section 2.0 of Appendix A and section 1.0 of Appendix B for the conditions outlined above. The results of these calculations appear in Table 1.

2.2 Lunar-Based Electronic and Laser Tracking Systems

The major difficulty in providing an accurate lunar-based tracking system to determine the relative position of an EVC is that line-of-sight between the LM and an EVC on a typical traverse out to 5 kilometers from the LM at a typical landing site cannot be guaranteed even with the use of long booms(1) because of obstructions caused by the irregular lunar terrain coupled with the effects of the general curvature of the Moon's surface. Possible solutions to this obstruction problem in determining the position of an EVC with respect to the LM, other than the use of Earth-based tracking systems, are to use (a) one or more portable-deployable line-of-sight tracking systems which could be set up to track the EVC when out of line-of-sight of the LM and to relay the tracking information to the LM for processing, or (b) LM-centered tracking systems which can make use of the property of ground wave radio propagation beyond the lunar horizon in the 100 kHz to 10 MHz frequency band discussed by Schmid.(7) These potential solutions are discussed in the following paragraphs.

2.2.1 Line-of-Sight Relay Tracking Systems

For this case, a transponder carried by an EVC would be tracked by a LM-centered tracking system until its line-of-sight to the transponder antenna was obstructed. Prior to losing line-of-sight of the LM, the EVC would deploy a portable tracking system package which could maintain line-of-sight of the crewman during portions of those periods when the crewman was not within line-of-sight of the LM. The tracking information gathered on the position of the transponder carried by crewman with respect to the portable tracking system package coordinates would be transmitted to the LM for processing via a line-of-sight RF link or via a ground wave RF link. The position of the portable tracking system package deployed by the crewman would be determined by the LM-centered tracking system. Prior to losing line-of-sight of the deployed portable tracking system package while remaining out of sight of the LM, a second portable tracking system package could be deployed by the EVC. The position of the second deployed portable tracking system package would be determined through use of the first deployed portable tracking system package. The tracking information on the movements of the EVC would be transmitted to the LM for processing via a line-of-sight RF relay link through the first deployed portable tracking system package or via a direct ground wave RF link. Any number of portable tracking

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(7) K. H. Schmid, "Required Transmitter Power for Ground Wave Radio Propagation Beyond the Lunar Horizon in the 100 kHz to 10 MHz Frequency Band," Memorandum for File, July 23, 1969.

systems could be similarly deployed. The number of and the frequency allocations for the data links and the tracking links to avoid radio frequency interference have not been considered by the writer nor have the operational procedures for such a system.

The only type of tracking system considered for use in this application were spherical coordinate tracking systems (relative position given in terms of slant range and two orthogonally related angles, frequently azimuth and elevation). Both RF and laser tracking systems were considered. Use of frequencies as high as possible are attractive to permit reduction of the size of the antenna of the tracking system while maintaining commensurate angle measurement accuracy.

The X-band rendezvous radar of the LM of the Apollo Program could be used to track an EVC equipped with an X-band rendezvous radar transponder until line-of-sight is lost. The slant range and angle measurement accuracies, including both bias and random error contributions, for the X-band rendezvous radar are specified at 10 meters and 10 milliradians, respectively. A laser radar could be used to track a retro-reflector carried by the EVC on a staff (Lunar Surveying System). The slant range and angle measurement accuracies quoted for the Lunar Surveying Systems are 0.5 meters and 0.5 degrees, respectively.

The predicted uncertainty in the determination of the position of the EVC in an arbitrary right-hand rectangular coordinate system centered at the LM using the spherical coordinate tracking systems described above was calculated using the error equations in section 3.0 of Appendix B for the case when the crewman is within line-of-sight of the LM at ranges of 2 and 5 kilometers for azimuth and elevation angles of 45 degrees and -5 degrees, respectively. The results of these calculations appear in Table 2. It is assumed that the location of the LM and the attitude of the LM and, as a consequence, the reference coordinate system will be precisely known. Using these values for uncertainty in the location of the first portable tracking system package deployed approximately 2 kilometers from the LM and assuming the accuracy in aligning the attitude of the tracking platform of the package was 1.0 degree with respect to the reference coordinate system of the LM in both of the orthogonal angles, the predicted uncertainty in the determination of the position of the EVC in the LM-centered arbitrary coordinate system was calculated using the error equations in section 3.0 of Appendix B for the case when the crewman is within line-of-sight of the first deployed portable tracking system package at a range of 2 kilometers for azimuth and elevation angles of -45 degrees and -5 degrees, respectively. The results of these calculations appear in Table 2. The results of similar calculations also appear in Table 2 for the case when the crewman is within line-of-sight of the second deployed portable tracking system package at a range of 2 kilometers for azimuth and elevation angles of 45 degrees and -5 degrees, respectively.

2.2.2 Ground Wave Tracking Systems

As a consequence of using ground wave radio propagation for transmission of tracking system signals to a remote receiver or transponder, elevation angle cannot be measured by a single tracking station. Therefore, use of a spherical coordinate tracking system located at the LM for determining the relative position of the EVC as discussed in the previous section is not possible when operating at frequencies in the 100 kHz to 10 MHz band. However, azimuth angle can be measured using conventional RF direction finding techniques currently employed on Earth. The accuracy of the azimuth angle measurement will be relatively gross, probably no better than 2 to 3 degrees. This order of accuracy would be sufficient to enable an EVC to navigate to an area within line-of-sight of the LM or of another EVC but would not be very useful in determining the precise position of the crewman with respect to the LM.

An alternative to the use of a tracking system which depends upon the measurement of angles is to use a tracking system which depends solely upon the measurement of distance and/or the measurement of the difference between two distances. To define the position of a point in three dimensions, slant ranges from three points of known position to the unknown point or the difference of slant ranges (or sum of slant ranges) from two points of known position to the unknown point for three pairs of points (three points are sufficient to provide three different pairs) of known position are required. It should be noted that the accuracy in the determination of the position of the unknown point depends upon the size of the baselines, the distance between the unknown point and the baseline, the location of the point compared to the "line-of-shoot" of the system, and the uncertainty in the measurement of slant range, sum of two slant ranges or difference of two slant ranges.

Results of a preliminary calculation of the position of an EVC on the lunar surface using distance data from three points of known position on the lunar surface using the error equations in section 4.1 of Appendix B indicated that the uncertainty in the height of the EVC above (or below) the plane defined by the three points of known position was so large

(770 meters) that determination of this height was meaningless. Unfavorable geometry of the three points of known position with respect to the EVC position was the reason for this large uncertainty. For this calculation, it was assumed that:

(a) the LM was the central one of three stations on the lunar surface forming two perpendicular baselines.

(b) the two stations other than the LM were deployed by an EVC 500 meters from the LM forming two perpendicular baselines within the accuracy of the laser radar of the Lunar Surveying System from the LM.

(c) the EVC to be located was 5 kilometers from the LM on a line bisecting the angle between the two baselines.

(d) the accuracy of the distance measurements from each station to the EVC was 10 meters.

(e) the distance measurements could be converted to slant range data while maintaining an accuracy of 10 meters.

The results of this calculation are included in Table 2.

As a consequence of this finding, it appeared logical to assume that the EVC was on a surface of a geometrical solid which approximates that portion of the surface of the Moon which is of interest. This assumption is made by ships at sea on Earth when using such systems as Loran and Raydist as navigation aids. It must be possible to determine a direct expression for the coordinates of a line of position provided by the tracking system on any geometrical solid chosen to approximate the surface of the Moon so that the position of a transponder carried by the EVC can be uniquely defined from the intersection of two lines of position. For convenience and ease of derivation of error equations for various types of tracking systems, that portion of interest of the surface of the Moon was approximated by a plane in this memorandum.

Two types of tracking systems were briefly examined under the conditions of the approximation discussed in the previous paragraph; namely, (a) the measurement of distance from two points of known position to the EVC and (b) the measurement of the difference in distance from two points of known position to the EVC for two pairs of points of known position. It should be noted that all points in a plane which are a given distance from a fixed point in that plane make up a curve which is a circle. All points in a plane at which the difference between the distances to two fixed points of known location are the same make up a curve which is a hyperbola. The intersection of any two such curves coupled with some a priori knowledge of the general location of the point of interest to resolve location ambiguities will define the position of that point.

It should be noted that a minimum of three stations of known location on the lunar surface are required to provide sufficient data to determine the position of the EVC if the "difference in distance" or hyperbolic tracking system is used while a minimum of two stations is required if a "distance measurement" tracking system is used. Since the expected position determination accuracies of the two systems are roughly comparable, further consideration of the hyperbolic tracking system was stopped because of the extra complexity introduced by the required deployment by a crewman of a second station. The LM, of course, would serve as one of the stations in either of these tracking systems.

In the system considered, measurements of distance from the LM to the EVC and from a second station deployed on the lunar surface to the crewman are required simultaneously. Hence, the EVC would be required to carry two transponders, one compatible with the operating frequencies of the LM station and the other compatible with the operating frequencies of the second station in order to avoid radio frequency interference problems. It is anticipated that the accuracy of the distance measurement could be as good as 10 meters if the phase of the transmitted and received carrier frequencies are compared at the stations to provide the fine grain distance data. However, ground wave radio propagation delay anomalies resulting from improper prediction of the electrical constants of the lunar surface or from the ground wave traversing mountains and valleys and delay instabilities in the transponders and other circuitry are potential error sources which must be better understood if this tracking system were to be adopted. Since the speed of the EVC is small, any uncertainties in time synchronization between the LM station and the second station will have negligible effect on the determination of the position of the EVC.

It was assumed that the laser radar of the Lunar Surveying System on the LM will be used to determine the position of the second station deployed by a crewman. Therefore, the second station was assumed to be located 500 meters from the LM with an uncertainty of 0.5 meters in the radial direction and 0.5 degrees in azimuth angle with respect to the LM-centered reference rectangular coordinate system whose orientation was assumed to be precisely known.

The predicted uncertainty in the determination of the position of an EVC with respect to the LM using two distance measurements from stations on the lunar surface was calculated using the error equations in section 4.2 of Appendix B under the conditions described above when the crewman is 5 kilometers from the LM and equidistant from the LM and the second station. The uncertainty in the relative position determination resulting from the error in measuring the azimuth angle of the location of the second station was added to the uncertainty calculated as described above on a root-sum-square basis to obtain to total predicted uncertainty. These calculations were repeated assuming the second station was located 1000 meters from the LM rather than 500 meters while keeping all other conditions constant. The results of these calculations appear in Table 2. It should be noted that this location of the crewman is on the "line-of-shoot" of this tracking system configuration where the accuracy will be the best. For locations of the crewman greater than 60 degrees from the "line-of-shoot" of this tracking system, the uncertainty in position determination blows up rapidly.

If the sum of the distances from the LM to the EVC and from the EVC back to the LM and the sum of the distances from the LM to the EVC and from the EVC to the second station were measured instead of the distances as discussed above, the EVC would be required to carry only one transponder. In this case, transmissions would emanate from the LM and be received, turned-around, and retransmitted by the transponder and RF system carried by the crewman to the LM and to the second station simultaneously. Reference signals could be transmitted from the LM to the second station for use in determining the distance sum from the LM to the EVC to the second station or the signal received by the second station could be relayed to the LM for determination of the distance sum. Although the differential change in delay after calibration between the two transponders carried by the crewman is eliminated as an error source in this technique because only one transponder is used, any drift in the absolute delay in the communications link (including all circuits) between the LM and the second station after calibration will be added as an error source. It was assumed that the standard deviation of the measurements of the sum of two distances would be 10 meters. It should be noted that, to measure the sum of two distances with the same accuracy as a distance measurement is made, the uncertainty in the delay through a transponder which is used in both measurements must be halved for the distance sum measurements, with all other things being equal. The predicted uncertainty in the determination of relative position of an EVC and the LM using two distance sum measurements by stations on the lunar surface was calculated using the error equations in Section 4.3 of Appendix B under the conditions described above when the second station is 500 meters from the LM and the crewman is 5 kilometers from both the LM and the second station. The results of these calculations appear in Table 2.

3.0 SUMMARY

The results of the calculations made to estimate the standard deviation of the uncertainty in determining the position of an extravehicular crewman with respect to the LM on the lunar surface appear in Table 1 for the Earth-based electronic tracking system configurations investigated and in Table 2 for the lunar-based tracking systems. The advantages and disadvantages of the methods investigated for relative position determination as well as a description of the methods are summarized in Table 3. In all cases it was assumed that the location and the orientation of the LM on the lunar surface could be determined independently and would be precisely known. The calculations on all of the Earth-based electronic tracking system configurations investigated were based on the use of the USB stations of the MSFN located at Ascension, Bermuda, and Goldstone, a fixed location of the LM, and a fixed location of the EVC with respect to the LM. The LM on the lunar surface was assumed to be equidistant from these three MSFN stations while the EVC was assumed to be positioned on the lunar surface at a slant range from the LM of 5 kilometers. As expected and shown in Table 1, the determination of the value of that coordinate of the relative position of the EVC with respect to the LM corresponding approximately to the difference in slant range from the MSFN stations to the LM and the EVC will be significantly more accurate than the determination of the value of the cross slant range coordinates because of the non-optimum geometry between the tracking stations on Earth and the transponders carried by the LM and EVC on the Moon.

Of the four Earth-based tracking system configurations investigated, the use of slant range sum data obtained through integration of Doppler frequency counts is the most attractive from the standpoints of minimizing equipment impact to the existing or planned capabilities of the stations of the MSFN and of minimizing the amount and complexity of equipment which must be carried by an EVC. No modifications would be required at any MSFN stations provided with a dual tracking capability. The EVC would be required to carry only one coherent S-band transponder (plus spares) and an RF receiving and transmitting system. The RF transmitting system could be low power because only an unmodulated continuous wave carrier would need to be transmitted. At a cost of increased effective radiated power requirements and some increased equipment complexities, this link could also be used as a direct link for voice communications with and telemetry transmission to stations of the MSFN. Relative position determination uncertainties of the order of 280 meters total are possible if integration of the random error of Doppler frequency counts over relatively long periods of time does not exceed 1 meter and if bias errors in the measurement of Doppler frequency counts at each station are approximately the same for both the LM and the EVC channels. It is clear that the types of and/or statistics of bias errors and random errors in slant range or slant range sum measurements made by USB stations of the MSFN using the pseudo-noise ranging capability or through integration of Doppler frequency counts obtained using the range rate measuring capability must be determined before a more reliable estimate of the relative position determination uncertainty can be made. It is clear, however, that the standard deviation of the error on the measurement of slant range, slant range sum, difference of slant ranges, or difference of slant range sums by Earth-based stations must be on the order of one meter or less for the use of the Earth-based tracking system configurations investigated to be attractive in this application.

Although the Earth-based tracking system configuration where one slant range measurement and two difference measurements of slant range are made appears attractive from the standpoint of the magnitude of the uncertainty in relative position determination, implementation may prove troublesome because of problems of frequency allocation for the necessary wideband channels, equipment impact at existing stations of the MSFN, and maintenance of long term (up to 4 hours after calibration) delay variation under 0.3 nanoseconds in the applicable equipments carried by the LM and the EVC and in the stations of the MSFN which are included in the channels of the LM and the EVC, respectively.

Of the lunar-based electronic and laser tracking systems investigated, line-of-sight tracking of the EVC using portable spherical coordinate tracking system packages from which data would be transferred to the LM appears to offer the most accurate method for determining the relative position of the EVC with respect to the LM. However, major problems do exist including (a) the limitation on the weight and volume of a load which an EVC can transport on the lunar surface in addition to that required for scientific and life support reasons and (b) the design of a rugged, light weight, and small package providing automatic acquisition and tracking of a target using the necessarily narrow beamwidth over ranges extending from approximately 1 meter to 5 kilometers. The over-the-horizon lunar-based electronic tracking systems investigated for use only when the LM and the EVC are not within line-of-sight do not offer significant improvement in the accuracy of relative position determination of the EVC over that provided by Earth-based tracking system configurations examined unless long baseline(s) are used. Thus one of the traverses of an EVC would have to be spent deploying and accurately locating remote tracking station(s). Furthermore, if only one baseline were used, it must be possible to approximate closely that portion of the lunar surface of interest with the surface of a geometrical solid on which the lines of position provided by the tracking system can be defined by explicit equations in order to maintain reasonable relative position determination accuracy. Although use of ground wave propagation of low frequency signals for precision tracking of the EVC does not appear very attractive; it does appear attractive if used for direction finding purposes to enable an EVC to return to within line-of-sight of the LM or to locate a second EVC who may be in trouble.

From the results of this cursory study, none of the methods investigated for determination of the relative position of an EVC with respect to the LM appears especially attractive. Of those methods investigated, the most attractive from the viewpoints of accuracy and minimum impact to existing systems and capabilities is the use of slant range sum data derived from integration of Doppler frequency counts by USB MSFN stations in a trilateration solution. However, the bias and random error statistics on slant range sum measurements made in this manner including the effects of EVC movements other than translational (twisting, bending, etc.) must be determined to verify the predicted accuracy of this method for relative position determination.

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Attachment
Tables 1 thru 3
Appendices A and B

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TABLE 1

ERROR IN RELATIVE POSITION DETERMINATION OF LM AND EXTRAVEHICULAR CREWMAN USING EARTH-BASED ELECTRONIC TRACKING SYSTEMS

Tracking System* Data	Standard Deviation of Position Determination Error (Meters)			
	ΔX_s	ΔY_s	ΔZ_s	Total
1. Three Slant Ranges				
(a) ΔR = 1 meter	115	252	3	278
(b) ΔR = 15 meters	1725	3780	46	4170
2. Three Slant Range Sums				
(a) ΔRS = 1 meter	116	256	3	282
(b) ΔRS = 15 meters	1745	3840	46	4230
3. Three Differences of Slant Range Sums				
(a) ΔDRS = 1.4 meters	191	172	1	257
(b) ΔDRS = 21 meters	2850	2580	8	3850
4. One Slant Range Plus Two Difference of Slant Ranges ΔD = 0.1 meter				
(a) ΔR = 1 meter	17	31	2	39
(b) ΔR = 15 meters	17	31	21	41

*Stations located at Ascension, Bermuda, and Goldstone

TABLE 2

ERROR IN RELATIVE POSITION DETERMINATION OF LM AND EXTRAVEHICULAR CREWMAN USING LUNAR-BASED TRACKING SYSTEMS

Tracking System Data	Standard Deviation of Position Determination Error (Meters)			
	ΔX_s	ΔY_s	ΔZ_s	Total
1. Slant Range Plus Two Orthogonal Angles				
(a) One 5 km Hop				
(1) Rendezvous Radar	36.6	36.6	49.8	71.9
(2) Laser	31.4	31.4	43.5	62.1
(b) One 2 km Hop				
(1) Rendezvous Radar	15.8	15.8	19.9	30.0
(2) Laser	12.4	12.4	17.4	24.6
(c) Two 2 km Hops				
(1) Rendezvous Radar	33.6	33.6	45.1	65.5
(2) Laser	30.2	30.2	42.5	60.3
(d) Three 2 km Hops				
(1) Rendezvous Radar	44.7	44.7	60.6	87.5
(2) Laser	40.9	40.9	57.6	81.5
2. Three Distances	133.1	166.0	770	
3. Two Distances				
(a) 500 Meter Baseline	148.0	7.4	-	148.2
(b) 1000 Meter Baseline	82.7	8.4	-	83.1
4. Two Distance Sums	148.0	7.4	-	148.2

TABLE 3

SUMMARY OF CHARACTERISTICS OF METHODS FOR DETERMINATION OF RELATIVE POSITION OF AN EVC WITH RESPECT TO LM

Method	Operation	Advantages	Disadvantages
1. Earth-Based Tracking Systems			
(a) Three Slant Range Measurements (Section 2.1.1)	• Three USB stations required. • Transponders required on EVC and LM for tracking aid. • Positions of LM and EVC determined simultaneously in same coordinate system. • Position of LM (or EVC) determined from slant range measurements made simultaneously by three stations. • Relative position of EVC with respect to LM determined from absolute positions of LM and EVC.	• USB transponder and RF system of EVC used for telemetry to and voice communications with MSFN.	• Line-of-sight • Direct MSFN/EVC communications required. • Three USB transponders and RF system carried by EVC. • Two additional USB transponders required on LM. • Major augmentation of the LM transmission capabilities required. • Frequency band utilization and allocation. • Major augmentation of the MSFN station capabilities required.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
(b) Three Slant Range Sum Measurements (Section 2.1.2)	• Three USB stations required. • Transponders required on EVC and LM for tracking aid. • Positions of LM and EVC determined simultaneously in same coordinate system. • Position of of LM (or EVC) determined from three slant range sum measurements made simultaneously. •• S-band transmissions from station "A" received, turned-around and re-transmitted by LM. •• S-band transmissions by LM received by stations "A", "B" and "C". •• After calibration, subsequent slant range sum(s) determined through delay measurements. • Relative position of EVC with respect to LM determined from absolute positions of LM and EVC.	• No additional transponder(s) required on the LM. • No impact on the MSFN. • USB transponder and RF system of EVC used for telemetry to and voice communications with MSFN.	• Line-of-sight required. • Direct MSFN/EVC communications required.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
(c) Three Difference Measurements of Slant Range Sums (Section 2.1.3)	• Three USB stations required. • Transponders required on EVC and LM for tracking aid. • Relative position of EVC with respect to LM determined directly from measurements of the difference between the slant range sum from one USB station to the LM and back to any USB station and the corresponding slant range sum to an EVC using the same stations for three combinations of stations simultaneously. •• S-band transmissions from station "A" received, turned-around, and retransmitted by EVC and LM. •• S-band transmissions from EVC and LM received by stations "A", "B" and "C". •• Difference of slant range sums determined from relative delay measurements.	• No additional transponder(s) required on LM. • USB transponder and RF system of EVC used for telemetry to and voice communications with MSFN.	• Line-of-sight required. • Direct MSFN/EVC communications required. • One USB transponder and RF system carried by EVC. • Slight augmentation of MSFN station capabilities required.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
(d) One Slant Range Measurement and Two Difference Measurements of Slant Range (Section 2.1.4)	• Three USB stations required. • Transponders required on EVC and LM for tracking aid. • Digital sequence generators required on EVC and LM. • Positions of LM and EVC determined simultaneously in same coordinate system. • Position of LM (or EVC) determined from one slant range measurement and two difference measurements of slant range made simultaneously. •• Slant range to LM measured by station "A". •• Digital sequence generated on-board LM and transmitted to stations "A", "B" and "C". •• Difference in slant range from LM to stations "A" and "B" and difference in slant range from LM to stations "A" and "C" determined. •• After calibration, difference in slant range determined from relative delay measurements. • Relative position of EVC with respect to LM determined from absolute positions of EVC and LM.	• USB transponder and RF system of EVC used for telemetry to and voice communications with MSFN.	• Line-of-sight required. • Direct MSFN/EVC communications required. • One USB transponder, a digital sequence generator and RF system carried by EVC. • Digital sequence generator carried by LM. • Major augmentation of LM transmission capabilities required. • Frequency band utilization and allocation. • Major augmentation of the MSFN station capabilities required. • Tight long term (4 hours) delay variation requirements for each channel.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
2. Lunar-Based Tracking Systems (a) Line-of-Sight Relay (Section 2.2.1)	• Transponder required on EVC for tracking aid. • Relative position of transponder with respect to the tracking system equipment determined by simultaneous measurement of slant range and two or thogonally related angles. • Tracking system equipment provided on the LM used until transponder no longer within line-of-sight. • Portable tracking system package(s) deployed by EVC to track transponder when not in line-of-sight of LM. • Measurements transmitted from portable tracking system package(s) to LM for processing to determine relative position of EVC with respect to LM.	• Direct MSFN/EVC communications not required.	• Line-of-sight required. • Transportation and deployment by EVC of portable tracking system package(s). • Design of small package providing automatic acquisition and tracking at ranges from 1 meter to 5 kilometers. • Complicated operations for tracking system management.

TABLE 3 (Continued)

Method	Operation	Advangtages	Disadvantages
(b) Ground Wave Tracking Systems (Section 2.2.2)			
(1) Three Distance Measurements	• Three stations required with the LM serving as the central station and performing the necessary data processing. • Transponders required on the EVC for tracking aid. • Relative position of FVC with respect to LM determined from distance measurements made simultaneously by the three stations. • Distance measurement results assumed equivalent to slant ranges.	• Line-of-sight not required. • EVC use tracking signals transmitted from LM for direction finding purposes.	• Three transponders carried by EVC. • Two stations deployed by EVC at least 500 meters from LM. • Accuracy of relative altitude determination severely limited by poor geometry of station locations. • Accuracy of relative position determination a strong function of location of EVC with respect to the baseline formed by the stations. • Distance measurement results not equivalent to slant ranges. • Ground wave propagation uncertainties.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
(2) Two Distance Measurements	• Two stations required with LM serving as the central station and performing the necessary processing of data. • Transponders required on the EVC for tracking aid. • Relative position of EVC with respect to LM determined from distance measurements made simultaneously by two stations coupled with an assumption of the geometrical shape of the local lunar surface.	• Line-of-sight not required. • EVC use tracking signals transmitted from LM for direction finding purposes.	• Two transponders carried by EVC. • One station deployed by EVC at least 500 meters from LM. • Accuracy of relative position determination a strong function of location of EVC with respect to the baseline. • Ground wave propagation uncertainties.

TABLE 3 (Continued)

Method	Operation	Advantages	Disadvantages
(3) Two Distance Sum Measurements	• Two stations required with the LM serving as the control station and performing the necessary data processing. • Transponder required on the EVC for tracking aid. • Relative position of EVC with respect to LM determined from two distance sum measurements made simultaneously coupled with an assumption of the geometrical shape of the local lunar surface. ••RF transmission from LM received, turned-around and retransmitted from EVC. ••RF transmission from EVC received by LM and other station.	• Line-of-sight not required. • One transponder carried by EVC. • EVC use tracking signals transmitted from LM for direction finding purposes.	• One station deployed by EVC at least 500 meters from LM. • Accuracy of relative position determination a strong function of location of EVC with respect to the baseline. • Ground wave propagation uncertainties.

APPENDIX A

POSITION DETERMINATION

1.0 USING TRILATERATION TECHNIQUES

1.1 Three Slant Ranges

For simplicity in deriving the equations describing the position of a point (space vehicle) in terms of the distance from three different points (sites) of known position, an arbitrary right-handed rectangular coordinate system (see Figure A-1) is defined with the origin located at the known position of point 1 (site 1). The positive X-axis is defined as the line in the direction from point 1 (origin) through point 2 (site 2). The XY plane is defined as the plane containing the X axis and point 3 (site 3). The X axis is defined as the line perpendicular to the XY plane and passing through site 1, positive in the direction which makes the Z coordinate of the location of the space vehicle positive. The positive Y axis is defined as that required to complete a right-handed rectangular coordinate system. The coordinates of the sites and the space vehicle in the arbitrary coordinate system will then be:

Site 1: $X = 0,\ Y = 0,\ Z = 0$

Site 2: $X = X_2,\ Y = 0,\ Z = 0$

Site 3: $X = X_3,\ Y = Y_3,\ Z = 0$

Space Vehicle, P: $X = X_s,\ Y = Y_s,\ Z = Z_s$

The scalar distance (or slant range) from the space vehicle to site 1 is defined as R_1; to site 2,as R_2; and to site 3, as R_3.

Equations were derived which define the position of the space vehicle in the arbitrary coordinate system in terms of the relative geocentric locations of the three fixed sites and the range from each site to the space vehicle. The equations are:

$$X_s = \frac{1}{2X_2}\left(R_1^2 - R_2^2 + X_2^2\right)$$

$$Y_s = \frac{1}{2Y_3}\left[R_1^2 - R_3^2 + X_3^2 + Y_3^2 - \frac{X_3}{X_2}\left(R_1^2 - R_2^2 + X_2^2\right)\right]$$

$$Z_s = \left(R_1^2 - X_s^2 - Y_s^2\right)^{1/2}$$

APPENDIX A

It was assumed that the uncertainties in the slant range measurements and the site geocentric locations are mutually independent. Thus the overall position determination uncertainty in each coordinate may be computed as the square root of the sum of the squares of the uncertainty caused by each error source alone. For this method of position determination, the equations which yield the magnitude of the uncertainty of the determined position are:

$$\Delta X_s = \left\{ \left[\left(\frac{X_2^2 + R_2^2 - R_1^2}{2X_2^2} \right) \Delta X_2 \right]^2 + \left[\left(\frac{R_1}{X_2} \right) \Delta R_1 \right]^2 + \left[\left(-\frac{R_2}{X_2} \right) \Delta R_2 \right]^2 \right\}^{1/2}$$

$$\Delta Y_s = \Bigg\{ \left(\left[\frac{Y_3^2 - R_1^2 + R_3^2 - X_3^2}{2Y_3^2} + \frac{X_3 \left(R_1^2 - R_2^2 + X_2^2 \right)}{2X_2 \, Y_3^2} \right] \Delta Y_3 \right)^2$$

$$+ \left[\left(\frac{R_1}{Y_3} - \frac{X_3 R_1}{X_2 Y_3} \right) \Delta R_1 \right]^2 + \left[\left(\frac{X_3 R_2}{X_2 Y_3} \right) \Delta R_2 \right]^2$$

$$+ \left[\left(\frac{X_3}{Y_3} - \frac{R_1^2 - R_2^2 + X_2^2}{2X_2 Y_3} \right) \Delta X_3 \right]^2$$

$$+ \left(\left[-\frac{X_3}{2Y_3} - \frac{X_3 \left(R_2^2 - R_1^2 \right)}{2X_2^2 Y_3} \right] \Delta X_2 \right)^2 + \left[\left(-\frac{R_3}{Y_3} \right) \Delta R_3 \right]^2 \Bigg\}^{1/2}$$

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$$\Delta Z_s = \left\{ \left(\left[R_1^2 - X_s^2 - Y_s^2 \right]^{-1/2} \left[R_1 - X_s \left(\frac{R_1}{X_2} \right) - Y_s \left(\frac{R_1}{Y_3} - \frac{X_3 R_1}{X_2 Y_3} \right) \right] \Delta R_1 \right)^2 \right.$$

$$+ \left(-X_s \left[R_1^2 - X_s^2 - Y_s^2 \right]^{-1/2} \left[\left(\Delta X_s \right)^2 - \left(\frac{R_1}{X_2} \Delta R_1 \right)^2 \right]^{1/2} \right)^2$$

$$\left. + \left(-Y_s \left[R_1^2 - X_s^2 - Y_s^2 \right]^{-1/2} \left[\left(\Delta Y_s \right)^2 - \left(\left[\frac{R_1}{Y_3} - \frac{X_3 R_1}{X_2 Y_3} \right] \Delta R_1 \right)^2 \right]^{1/2} \right)^2 \right\}^{1/2}$$

1.2 Three Slant Range Sums

Using the same right-handed rectangular coordinate system defined in the preceding section (see Figure A-1), the coordinates of the three sites and the space vehicle will also be:

Site 1: $X = 0,\ Y = 0,\ Z = 0$

Site 2: $X = X_2,\ Y = 0,\ Z = 0$

Site 3: $X = X_3,\ Y = Y_3,\ Z = 0$

Space Vehicle, P: $X = X_s,\ Y = Y_s,\ Z = Z_s$

Assuming that site 1 is the master station and that site 2 and site 3 are slave stations, the range sum from site 1 to the space vehicle and back to site 1 is defined as RS_1; the range sum from site 1 to the space vehicle to site 2 is defined as RS_2, and the range sum from site 1 to the space vehicle to site 3 is defined as RS_3.

Equations were derived which define the position of the space vehicle in the arbitrary coordinate system in terms of the relative geocentric locations of the three fixed sites and the three range sums defined above.

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The equations are:

$$X_s = \frac{1}{2X_2}\left[X_2^2 + (RS_1)(RS_2) - (RS_2)^2\right]$$

$$Y_s = \frac{1}{2Y_3}\left[X_3^2 + Y_3^2 + (RS_1)(RS_3) - (RS_3)^2 - \frac{X_3}{X_2}\left(X_2^2 + (RS_1)(RS_2) - (RS_2)^2\right)\right]$$

$$Z_s = \left[\left(\frac{RS_1}{2}\right)^2 - X_s^2 - Y_s^2\right]^{1/2}$$

Since the uncertainties in the range sum measurements and the site geocentric locations were assumed to be mutually independent, the overall uncertainty in each coordinate of the position determination of the space vehicle may be found from the following equations:

$$\Delta X_s = \left\{\left(\left[\frac{X_2^2 - (RS_1)(RS_2) + (RS_2)^2}{2X_2^2}\right]\Delta X_2\right)^2 + \left[\left(\frac{RS_1 - 2RS_2}{2X_2}\right)\Delta RS_2\right]^2 + \left[\left(\frac{RS_2}{2X_2}\right)\Delta RS_1\right]^2\right\}^{1/2}$$

$$\Delta Y_s = \left\{\left(\left[\frac{X_3}{Y_3} - \frac{X_2^2 + (RS_1)(RS_2) - (RS_2)^2}{2X_2Y_3}\right]\Delta X_3\right)^2 + \left(\left[\frac{RS_3}{2Y_3} - \frac{X_3(RS_2)}{2X_2Y_3}\right]\Delta RS_1\right)^2 + \left[\left(\frac{RS_1 - 2RS_3}{2Y_3}\right)\Delta RS_3\right]^2 + \left(X_3\left[\frac{X_2^2 - (RS_1)(RS_2) + (RS_2)^2}{2X_2^2Y_3}\right]\Delta X_2\right)^2\right.$$

$$+\left(\left[-\frac{X_3(RS_1-2RS_2)}{2X_2Y_3}\right]\Delta RS_2\right)^2$$

$$\left.+\left(\left[\frac{Y_3^2-X_3^2-(RS_1)(RS_3)+(RS_3)^2}{2Y_3^2}+\frac{X_3\left[X_2^2+(RS_1)(RS_2)-(RS_2)^2\right]}{2X_2Y_3^2}\right]\Delta Y_3\right)^2\right\}^{1/2}$$

$$\Delta Z_s=\left\{\left(\left[\left(\frac{RS_1}{2}\right)^2-X_s^2-Y_s^2\right]^{-1/2}\left[\frac{RS_1}{2}-X_s\left(\frac{RS_2}{2X_2}\right)-Y_s\left(\frac{RS_3}{2Y_3}-\frac{X_3(RS_2)}{2X_2Y_3}\right)\right]\Delta RS_1\right)^2\right.$$

$$+\left(-X_s\left[\left(\frac{RS_1}{2}\right)^2-X_s^2-Y_s^2\right]^{-1/2}\left[\left(\Delta X_s\right)^2-\left(\frac{RS_2}{2X_2}\Delta RS_1\right)^2\right]^{1/2}\right)^2$$

$$\left.+\left(-Y_s\left[\left(\frac{RS_1}{2}\right)^2-X_s^2-Y_s^2\right]^{-1/2}\left[\left(\Delta Y_s\right)^2-\left(\frac{RS_3}{2Y_3}-\frac{X_3(RS_2)}{2X_2Y_3}\right)^2\right]^{1/2}\right)^2\right\}^{1/2}$$

2.0 USING INTERFEROMETRY PRINCIPLES

In this method for position determination of a point (space vehicle), three different points (sites) of known position are required. One of the three sites serves as a central station and the slant range from this site to the space vehicle is measured. The difference in slant range from the space vehicle to the central station and to one of the two remaining sites and the difference in slant range from the space vehicle to the central station and to the other of the two remaining sites are measured simultaneously with the slant range measurement described above. The slant range difference measurement can be used to calculate the cosine of the angle between a line connecting the two sites (baseline) and a line connecting the space vehicle with the mid-point of the baseline. Since the distance between the space vehicle and any one of the three sites is very much larger than the length of the baselines used in determining the cosines of the angles, it was assumed that the angle defined by each cosine was approximately equal to the angle between the slant range from the central station to the spacevehicle and the respective baseline.

Using the same right-handed rectangular coordinate system defined in the preceding section (see Figure A-1), the coordinates of the three sites and space vehicle will remain:

Site 1: $X = 0,\ Y = 0,\ Z = 0$

Site 2: $X = X_2,\ Y = 0,\ Z = 0$

Site 3: $X = X_3,\ Y = Y_3,\ Z = 0$

Space Vehicle, P: $X = X_s,\ Y = Y_s,\ Z = Z_s$

It is assumed that site 1 would serve as the central station. The slant range from site 1 to the space vehicle is defined as R_1. The difference in slant ranges from the space vehicle to site 1 and to site 2 is defined as D_2 and the difference in slant ranges from the space vehicle to site 1 and to site 3 is defined as D_3.

Equations were derived which define the position of the space vehicle in the arbitrary coordinate system in terms of the relative geocentric locations of the three fixed sites, the slant range from the central station to the space vehicle, and the two slant range differences defined above. These equations are:

$$X_s = \frac{2D_2R_1}{X_2}$$

$$Y_s = \frac{2D_3R_1}{Y_3} - \frac{2X_3D_2R_1}{X_2Y_3}$$

$$Z_s = \left(R_1^2 - X_s^2 - Y_s^2\right)^{1/2}$$

Since the uncertainties in the slant range and slant range difference measurements and the site geocentric locations were assumed to be mutually independent, the overall uncertainty in each coordinate of the determined position of the space vehicle may be found from the following equations:

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$$\Delta X_s = \left\{ \left[\left(\frac{2D_2}{X_2} \right) \Delta R_1 \right]^2 + \left[\left(\frac{2R_1}{X_2} \right) \Delta D_2 \right]^2 + \left[\left(-\frac{2D_2 R_1}{X_2^2} \right) \Delta X_2 \right]^2 \right\}^{1/2}$$

$$\Delta Y_s = \left\{ \left[\left(\frac{2D_3}{Y_3} - \frac{2X_3 D_2}{X_2 Y_3} \right) \Delta R_1 \right]^2 + \left[\left(-\frac{2X_3 R_1}{X_2 Y_3} \right) \Delta D_2 \right]^2 + \left[\left(\frac{2R_1}{Y_3} \right) \Delta D_3 \right]^2 \right.$$

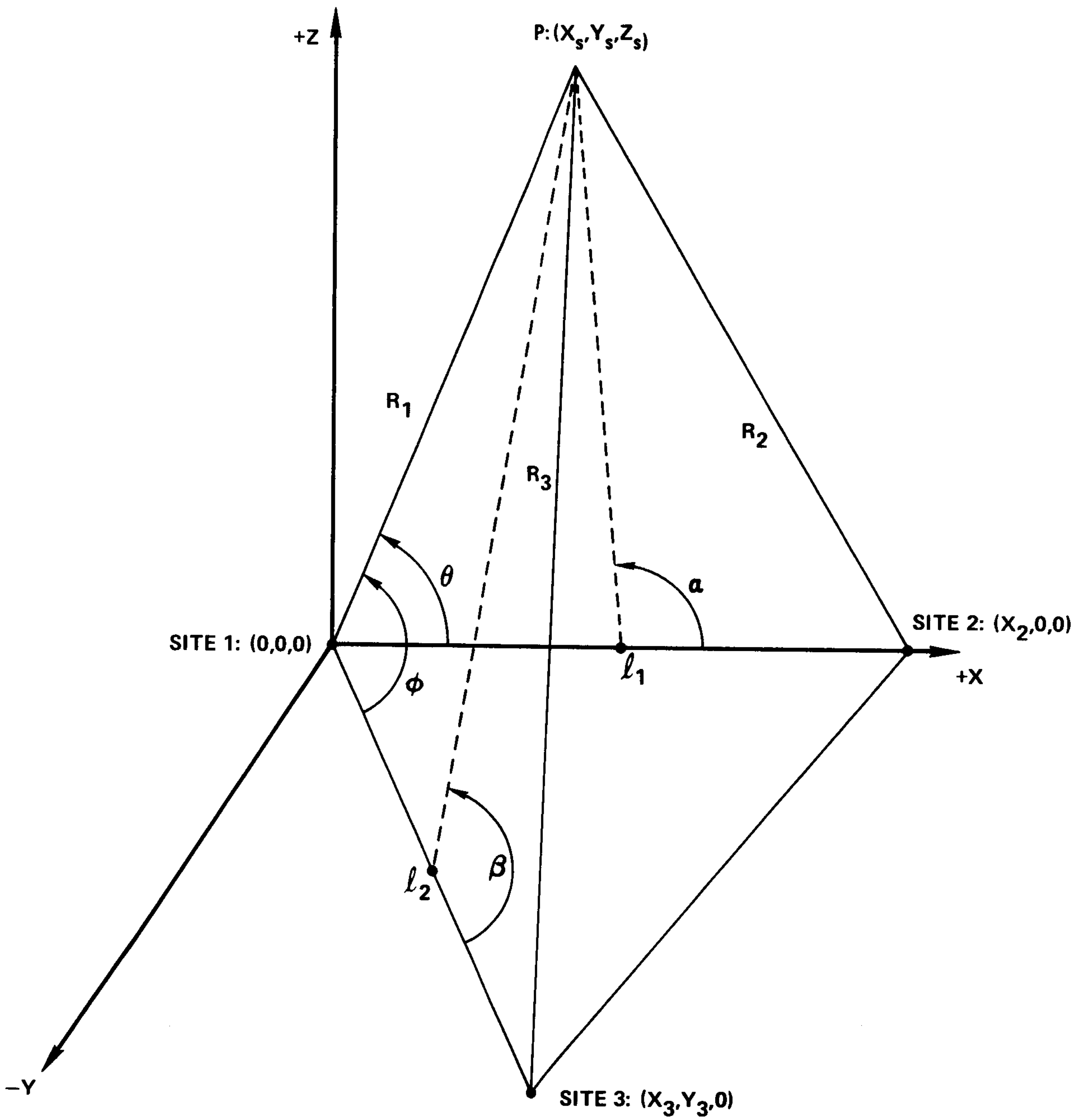
$$\left. + \left[\left(-\frac{2D_2 R_1}{X_2 Y_3} \right) \Delta X_3 \right]^2 + \left[\left(\frac{2X_3 D_2 R_1}{X_2^2 Y_3} \right) \Delta X_2 \right]^2 + \left[\left(-\frac{2D_3 R_1}{Y_3^2} + \frac{2X_3 D_2 R_1}{X_2 Y_3^2} \right) \Delta Y_3 \right]^2 \right\}^{1/2}$$

$$\Delta Z_s = \left\{ \left[\left(R_1^2 - X_s^2 - Y_s^2 \right)^{-1/2} \left(R_1 - X_s \left[\frac{2D_2}{X_2} \right] - Y_s \left[\frac{2D_3}{Y_3} - \frac{2X_3 D_2}{X_2 Y_3} \right] \right) \Delta R_1 \right]^2 \right.$$

$$+ \left(-X_s \left(R_1^2 - X_s^2 - Y_s^2 \right)^{-1/2} \left[\left(\Delta X_s \right)^2 - \left(\frac{2D_2}{X_2} \Delta R_1 \right)^2 \right]^{1/2} \right)^2$$

$$\left. + \left(-Y_s \left(R_1^2 - X_s^2 - Y_s^2 \right)^{-1/2} \left[\left(\Delta Y_s \right)^2 - \left(\left[\frac{2D_3}{Y_3} - \frac{2X_3 D_2}{X_2 Y_3} \right] \Delta R_1 \right)^2 \right]^{1/2} \right)^2 \right\}^{1/2}$$

It should be noted that these error equations do not include the uncertainty in the determination of the space vehicle position introduced by the assumptions that $\theta = \alpha$ and $\phi = \beta$ (see Figure A-1). The uncertainty resulting from this approximation is predictable and could be corrected for in the calculation of X_s, Y_s, and Z_s.



SINCE $R_1 >>> \ell_1$

$R_2 >>> \ell_1$

$R_1 >>> \ell_2$

$R_3 >>> \ell_2$

THEN $\theta \cong \alpha$

$\phi \cong \beta$

$2R_1 \triangleq RS_1$

$R_1 + R_2 \triangleq RS_2$

$R_1 + R_3 \triangleq RS_3$

$R_1 - R_2 \triangleq D_2$

$R_1 - R_3 \triangleq D_3$

FIGURE A-1

APPENDIX B

RELATIVE POSITION DETERMINATION

1.0 USING LOCATION OF TWO POINTS KNOWN IN AN ARBITRARY COORDINATE SYSTEM

Assuming that the locations of point A (LM) and point B (EVC) are known with respect to an arbitrary rectangular coordinate system such as could be achieved by using slant range or slant range sum trilateration techniques or interferometry techniques as described in Appendix A, the position of the EVC with respect to the LM could be defined in a rectangular coordinate system by translating the origin of the arbitrary rectangular coordinate system to the location of the LM.

It should be noted that the uncertainty in the site geocentric locations are bias errors and will be constant for a given site during all slant range or slant range sum measurements while the uncertainty in each slant range or slant range sum measurement is random and mutually independent. Generalized equations were derived defining the uncertainty in each of the three rectangular coordinates of the EVC with respect to the fixed LM for the case where the positions of both the EVC and the LM were determined in an arbitrary coordinate system using the slant range trilateration technique. These equations which are included below will also apply to the case where the positions of both the EVC and the LM were determined in an arbitrary coordinate system using the slant range sum trilateration technique if the parameters R_1, R_2, R_3 are replaced by RS_1, RS_2, and RS_3, respectively. LM peculiar location parameters are denoted by the subscript "s" while the EVC peculiar location parameters are denoted by the subscript "c".

$$\Delta X = \left\{ \left[\left(\frac{\partial X_c}{\partial X_2} - \frac{\partial X_s}{\partial X_2} \right) \Delta X_2 \right]^2 + \left[\frac{\partial X_c}{\partial \left| R_1 \right|_c} \Delta \left| R_1 \right|_c \right]^2 + \left[\frac{\partial X_c}{\partial \left| R_2 \right|_c} \Delta \left| R_2 \right|_c \right]^2 \right.$$
$$\left. + \left[\frac{\partial X_s}{\partial \left| R_1 \right|_s} \Delta \left| R_1 \right|_s \right]^2 + \left[\frac{\partial X_s}{\partial \left| R_2 \right|_s} \Delta \left| R_2 \right|_s \right]^2 \right\}^{1/2}$$

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$$\Delta Y = \left\{ \left[\left(\frac{\partial Y_c}{\partial Y_3} - \frac{\partial Y_s}{\partial Y_3}\right) \Delta Y_3\right]^2 + \left[\left(\frac{\partial Y_c}{\partial X_2} - \frac{\partial Y_s}{\partial X_2}\right) \Delta X_2\right]^2 + \left[\left(\frac{\partial Y_c}{\partial X_3} - \frac{\partial Y_s}{\partial X_3}\right) \Delta X_3\right]^2 \right.$$
$$+ \left[\frac{\partial Y_c}{\partial \left(R_1\right)_c} \Delta \left(R_1\right)_c\right]^2 + \left[\frac{\partial Y_c}{\partial \left(R_2\right)_c} \Delta \left(R_2\right)_c\right]^2 + \left[\frac{\partial Y_c}{\partial \left(R_3\right)_c} \Delta \left(R_3\right)_c\right]^2$$
$$\left. + \left[\frac{\partial Y_s}{\partial \left(R_1\right)_s} \Delta \left(R_1\right)_s\right]^2 + \left[\frac{\partial Y_s}{\partial \left(R_2\right)_s} \Delta \left(R_2\right)_s\right]^2 + \left[\frac{\partial Y_s}{\partial \left(R_3\right)_s} \Delta \left(R_3\right)_s\right]^2 \right\}^{1/2}$$

$$\Delta Z = \left\{ \left[\left(\frac{\partial Z_c}{\partial Y_3} - \frac{\partial Z_s}{\partial Y_3}\right) \Delta Y_3\right]^2 + \left[\left(\frac{\partial Z_c}{\partial X_2} - \frac{\partial Z_s}{\partial X_2}\right) \Delta X_2\right]^2 + \left[\left(\frac{\partial Z_c}{\partial X_3} - \frac{\partial Z_s}{\partial X_3}\right) \Delta X_3\right]^2 \right.$$
$$+ \left[\frac{\partial Z_c}{\partial \left(R_1\right)_c} \Delta \left(R_1\right)_c\right]^2 + \left[\frac{\partial Z_c}{\partial \left(R_2\right)_c} \Delta \left(R_2\right)_c\right]^2 + \left[\frac{\partial Z_c}{\partial \left(R_3\right)_c} \Delta \left(R_3\right)_c\right]^2$$
$$\left. + \left[\frac{\partial Z_s}{\partial \left(R_1\right)_s} \Delta \left(R_1\right)_s\right]^2 + \left[\frac{\partial Z_s}{\partial \left(R_2\right)_s} \Delta \left(R_2\right)_s\right]^2 + \left[\frac{\partial Z_s}{\partial \left(R_3\right)_s} \Delta \left(R_3\right)_s\right]^2 \right\}^{1/2}$$

A similar set of equations can be written to apply to the case where the positions of both the EVC and the LM were determined in an arbitrary coordinate system using an interferometry technique. For this case, the error in the site geocentric locations will be constant while the uncertainty in the slant range and slant range difference measurements will be random.

2.0 USING THREE DIFFERENCES BETWEEN TWO SETS OF THREE RANGE SUMS

A method for determining the relative position of point B (EVC - extravehicular crewman) with respect to point A (LM) using three differential range sum values obtained by subtracting the range sum from an arbitrary point (site) to the EVC to a second arbitrary point (site) from the similarly formed range sum between the same two fixed sites and the LM for three different sets of sites was proposed by James.[6]

For simplicity in deriving the equations defining the relative position of the EVC with respect to the LM using the method suggested by James, an arbitrary right-handed rectangular coordinate system is defined with the origin located at the position of the LM. The positive Z-axis is defined as the line positive in direction from the LM to the position of an arbitrary point designated site 1. The X-axis and Y-axis are chosen arbitrarily with the only restriction being that the reference coordinate system be rectangular in a right-handed sense. Then the coordinates of the sites, the LM, and the EVC used in this derivation (see Figure B-1) will be:

LM, A: $X = 0,\ Y = 0,\ Z = 0$

Site 1: $X = 0,\ Y = 0,\ Z = R$

Site 2: $X = X_2,\ Y = Y_2,\ Z = (R + Z_2)$

Site 3: $X = X_3,\ Y = Y_3,\ Z = (R + Z_3)$

EVC, B: $X = X_s,\ Y = Y_s,\ Z = Z_s$

Assuming that site 1 is the master station and that sites 2 and 3 are slave stations, the scalar difference determined by subtracting the range sum from site 1 to the EVC to site 1 from the range sum from site 1 to the LM to site 1 is defined as DRS_1, the scalar difference determined by subtracting the range sum from site 1 to the EVC to site 2 from the range sum from site 1 to the LM to site 2 is defined as DRS_2, and the scalar difference determined by subtracting the range sum from site 1 to the EVC to site 3 from the range sum from site 1 to the LM to site 3 is defined as DRS_3. The range sum from site 1 to the LM and back to site 1 is defined as RS_1.

Equations were derived which define the position of the EVC with respect to the LM in the arbitrary coordinate

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[6] D. B. James, "A Method of Navigating On and Near the Moon," Memorandum for File, June 28, 1968.

system in terms of the locations of the three fixed sites and the range sum and the three differences between range sums defined above. The derivation of these equations depends on the validity of the assumptions that $\frac{RS_1}{2} \gg$ distance between any two sites and that $\frac{RS_1}{2} \gg \left[X_s^2 + Y_s^2 + Z_s^2\right]^{1/2}$. Furthermore, the derivation of the equation for Z_s depends on the assumption that $RS_1 \gg X_s^2 + Y_s^2 + Z_s^2$. These equations are:

$$X_s = \frac{\left[Y_3\left(DRS_2 - DRS_1\right) - Y_2\left(DRS_3 - DRS_1\right)\right]\left(\frac{RS_1}{2}\right) + \left[Y_2 Z_3 - Y_3 Z_2\right]\left(\frac{DRS_1}{2}\right)}{\left[X_2 Y_3 - X_3 Y_2\right]}$$

$$Y_s = \frac{\left[X_3\left(DRS_2 - DRS_1\right) - X_2\left(DRS_3 - DRS_1\right)\right]\left(\frac{RS_1}{2}\right) + \left[X_2 Z_3 - X_3 Z_2\right]\left(\frac{DRS_1}{2}\right)}{\left[X_3 Y_2 - X_2 Y_3\right]}$$

$$Z_s = \frac{DRS_1}{2}$$

Since the uncertainties in the differential range sum measurements, the range sum measurements, and the site geocentric locations were assumed to be mutually independent, the overall uncertainty in each of the three rectangular coordinates of the position of the EVC respect to the fixed LM may be found from the following equations:

$$\Delta X_s = \left\{ \left[-Y_3 \left(\frac{\left[Y_3\left(DRS_2 - DRS_1\right) - Y_2\left(DRS_3 - DRS_1\right)\right]\left(\frac{RS_1}{2}\right) + \left[Y_2 Z_3 - Y_3 Z_2\right] Z_s}{\left[X_2 Y_3 - X_3 Y_2\right]^2} \right) \Delta X_2 \right]^2 \right.$$

$$+\left[+Y_2\left(\frac{\left[Y_3\left(DRS_2-DRS_1\right)-Y_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left[Y_2Z_3-Y_3Z_2\right]Z_s}{\left[X_2Y_3-X_3Y_2\right]^2}\right)\Delta X_3\right]^2$$

$$+\left[\left(\frac{\left[X_2Y_3-X_3Y_2\right]\left[-\left(DRS_3-DRS_1\right)\left(\frac{RS_1}{2}\right)+Z_3Z_s\right]}{\left[X_2Y_3-X_3Y_2\right]^2}\right.\right.$$

$$\left.\left.+\frac{X_3\left(\left[Y_3\left(DRS_2-DRS_1\right)-Y_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left[Y_2Z_3-Y_3Z_2\right]Z_s\right)}{\left[X_2Y_3-X_3Y_2\right]^2}\right)\Delta Y_2\right]^2$$

$$+\left[\left(\frac{\left[X_2Y_3-X_3Y_2\right]\left[\left(DRS_2-DRS_1\right)\left(\frac{RS_1}{2}\right)-Z_2Z_s\right]}{\left[X_2Y_3-X_3Y_2\right]^2}\right.\right.$$

$$\left.\left.-\frac{X_2\left(\left[Y_3\left(DRS_2-DRS_1\right)-Y_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left[Y_2Z_3-Y_3Z_2\right]Z_s\right)}{\left[X_2Y_3-X_3Y_2\right]^2}\right)\Delta Y_3\right]^2$$

$$+\left[\left(\frac{-Y_3Z_s}{X_2Y_3-X_3Y_2}\right)\Delta Z_2\right]^2+\left[\left(\frac{Y_2Z_s}{X_2Y_3-X_3Y_2}\right)\Delta Z_3\right]^2+\left[\left(\frac{Y_2Z_3-Y_3Z_2}{X_2Y_3-X_3Y_2}\right)\Delta Z_s\right]^2$$

$$+\left[\left(\frac{Y_3\left[\frac{RS_1}{2}\right]}{X_2Y_3-X_3Y_2}\right)\Delta\left(DRS_2-DRS_1\right)\right]^2+\left[\left(\frac{-Y_2\left[\frac{RS_1}{2}\right]}{X_2Y_3-X_3Y_2}\right)\Delta\left(DRS_3-DRS_1\right)\right]^2$$

$$\left.+\left[\left(\frac{Y_3\left[DRS_2-DRS_1\right]-Y_2\left[DRS_3-DRS_1\right]}{2\left[X_2Y_3-X_3Y_2\right]}\right)\Delta\left(RS_1\right)\right]^2\right\}^{1/2}$$

$$\Delta Y_s=\left\{\left[\left(\frac{\left[X_3Y_2-X_2Y_3\right]\left[-\left(DRS_3-DRS_1\right)\left(\frac{RS_1}{2}\right)+Z_3Z_s\right]}{\left[X_3Y_2-X_2Y_3\right]^2}\right.\right.\right.$$

$$\left.\left.+\frac{Y_3\left(\left[X_3\left(DRS_2-DRS_1\right)-X_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left(X_2Z_3-X_3Z_2\right)Z_s\right)}{\left[X_3Y_2-X_2Y_3\right]^2}\right)\Delta X_2\right]^2$$

$$+\left[\left(\frac{\left[X_3Y_2-X_2Y_3\right]\left[\left(DRS_2-DRS_1\right)\left(\frac{RS_1}{2}\right)-Z_2Z_s\right]}{\left[X_3Y_2-X_2Y_3\right]^2}\right.\right.$$

$$\left.\left.-\frac{Y_2\left(\left[X_3\left(DRS_2-DRS_1\right)-X_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left(X_2Z_3-X_3Z_2\right)Z_s\right)}{\left[X_3Y_2-X_2Y_3\right]^2}\right)\Delta X_3\right]^2$$

$$+\left[-X_3\left(\frac{\left[X_3\left(DRS_2-DRS_1\right)-X_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left(X_2Z_3-X_3Z_2\right)Z_s}{\left[X_3Y_2-X_2Y_3\right]^2}\right)\Delta Y_2\right]^2$$

$$+\left[+X_2\left(\frac{\left[X_3\left(DRS_2-DRS_1\right)-X_2\left(DRS_3-DRS_1\right)\right]\left(\frac{RS_1}{2}\right)+\left(X_2Z_3-X_3Z_2\right)Z_s}{\left[X_3Y_2-X_2Y_3\right]^2}\right)\Delta Y_3\right]^2$$

$$+\left[\left(\frac{-X_3Z_s}{X_3Y_2-X_2Y_3}\right)\Delta Z_2\right]^2+\left[\left(\frac{X_2Z_s}{X_3Y_2-X_2Y_3}\right)\Delta Z_3\right]^2+\left[\left(\frac{X_2Z_3-X_3Z_2}{X_3Y_2-X_2Y_3}\right)\Delta Z_s\right]^2$$

$$+\left[\left(\frac{X_3\left[\frac{RS_1}{2}\right]}{X_3Y_2-X_2Y_3}\right)\Delta\left(DRS_2-DRS_1\right)\right]^2+\left[\left(\frac{-X_2\left[\frac{RS_1}{2}\right]}{X_3Y_2-X_2Y_3}\right)\Delta\left(DRS_3-DRS_1\right)\right]^2$$

$$\left.+\left[\left(\frac{X_3\left(DRS_2-DRS_1\right)-X_2\left(DRS_3-DRS_1\right)}{2\left(X_3Y_2-X_2Y_3\right)}\right)\Delta\left(RS_1\right)\right]^2\right\}^{1/2}$$

$$\Delta Z_s = 1/2\ \Delta\left(DRS_1\right)$$

3.0 <u>USING RANGE AND TWO ANGLES MEASURED FROM ONE OF THE TWO POINTS</u>

An arbitrary right-handed rectangular coordinate system is used in the derivation of the position of a point (EVC) in terms of slant range, azimuth angle, and elevation angle data from a single fixed point (site or LM). The origin of the coordinate system is defined as the known location of the LM. The orientation of the X, Y, and Z axes is completely arbitrary providing that the right-handed rectangular properties of the coordinate system are maintained. The coordinates of the EVC and the LM as shown in Figure B-1 will then be:

$$\text{LM:} \quad X = 0,\ Y = 0,\ Z = 0$$

$$\text{EVC, P:} \quad X = X_s,\ Y = Y_s,\ Z = Z_s$$

The scalar distance or slant range from the LM to the EVC is defined as R. The azimuth angle, α, is defined as the angle measured counterclockwise from the X axis. The elevation angle, ε, is defined as the angle measured from the XY plane, positive in the direction of the positive Z axis.

Equations were derived which define the position of the EVC in the arbitrary coordinate system in terms of slant range, azimuth angle, and elevation angle from the LM or other site to the EVC. These equations are:

$$X_s = R \cos \varepsilon \cos \alpha$$

$$Y_s = R \cos \varepsilon \sin \alpha$$

$$Z_s = R \sin \varepsilon$$

Since the uncertainties in the slant range, azimuth angle, and elevation angle measurements were assumed to be mutually independent, the overall uncertainty in each of the three rectangular coordinates of the position of the EVC with respect to the fixed site from where the slant range and angle measurements were made may be found from the following equations:

$$\Delta Y_s = \left\{ \left[(\cos \varepsilon \sin \alpha) \Delta R \right]^2 + \left[(-R \sin \varepsilon \sin \alpha)\ \Delta\varepsilon \right]^2 + \left[(R \cos \varepsilon \cos \alpha)\ \Delta\alpha \right]^2 \right\}^{1/2}$$

$$\Delta X_s = \left\{ \left[(\cos \varepsilon \cos \alpha)\ \Delta R \right]^2 + \left[(-R \sin \varepsilon \cos \alpha)\ \Delta\varepsilon \right]^2 + \left[(-R \cos \varepsilon \sin \alpha)\ \Delta\alpha \right]^2 \right\}^{1/2}$$

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$$\Delta Z_s = \left\{ \left[(\sin \varepsilon)\ \Delta R \right]^2 + \left[(R \cos \varepsilon)\ \Delta \varepsilon \right]^2 \right\}^{1/2}$$

If the position of the EVC is given in terms of slant range, azimuth angle, and elevation angle measurements from a single fixed site different from the LM, then the coordinates of the fixed site, the EVC and the LM in the LM-centered right-handed rectangular coordinate system will be:

LM: $X = 0,\ Y = 0,\ Z = 0$

Fixed Site: $X = X_s',\ Y = Y_s',\ Z = Z_s'$

EVC: $X = X_s,\ Y = Y_s,\ Z = Z_s$

Maintaining the same nomenclature and definitions for slant range, azimuth angle, and elevation angle used earlier and assuming the axes of the reference coordinate system for measurement of azimuth and elevation angles at the LM and at the fixed site are parallel, equations were derived which define the position of the EVC in the arbitrary coordinate system in terms of slant range, azimuth angle, and elevation angle from the fixed site to the EVC and of the location of the fixed site with respect to the LM. These equations are:

$$X_s = R \cos \varepsilon \cos \alpha + X_s'$$

$$Y_s = R \cos \varepsilon \sin \alpha + Y_s'$$

$$Z_s = R \sin \varepsilon + Z_s'$$

Since the uncertainties in the slant range, azimuth angle, elevation angle, and position of the fixed site were assumed to be mutually independent, the overall uncertainty in each of the three rectangular coordinates of the position of the EVC with respect to the LM may be found from the following equations:

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$$\Delta Y_s = \left\{ \left[(\cos\ \varepsilon\ \sin\ \alpha)\ \Delta R\right]^2 + \left[(-R\ \sin\ \varepsilon\ \sin\ \alpha)\ \Delta\varepsilon\right]^2 + \left[(R\ \cos\ \varepsilon\ \cos\ \alpha)\ \Delta\alpha\right]^2 + \left[\Delta Y_s'\right]^2 \right\}^{1/2}$$

$$\Delta X_s = \left\{ \left[(\cos\ \varepsilon\ \cos\ \alpha)\ \Delta R\right]^2 + \left[(-R\ \sin\ \varepsilon\ \cos\ \alpha)\ \Delta\varepsilon\right]^2 + \left[(-R\ \cos\ \varepsilon\ \sin\ \alpha)\ \Delta\alpha\right]^2 + \left[\Delta X_s'\right]^2 \right\}^{1/2}$$

$$\Delta Z_s = \left\{ \left[(\sin\ \varepsilon)\ \Delta R\right]^2 + \left[(R\ \cos\ \varepsilon)\ \Delta\varepsilon\right]^2 + \left[\Delta Z_s'\right]^2 \right\}^{1/2}$$

4.0 USING ONE OF THE TWO POINTS AS THE CENTRAL STATION OF A BASELINE TRACKING SYSTEM

4.1 Three Ranges

In this case, it is assumed that the one fixed point (LM) would serve as a central station and be used with two slave stations to form two baselines. The coordinate system used in the derivation of the equations describing the location of a point (EVC) with respect to the LM is a right-handed rectangular coordinate system with the origin located at the LM or central station. The positive X-axis is defined as the line in the direction from the LM to one of the two slave stations. The XY plane is defined as the plane containing the X-axis and the second of the two slave stations. The positive Z-axis is defined as the line passing through the LM location, perpendicular to the XY plane, in the direction away from the center of the Moon. The parameter definitions and the equations derived in Section 1.1 of Appendix A for position determination as well as for the uncertainty of this determination using trilateration with slant range measurements from three known points to the unknown point will also apply to this case.

4.2 Two Distances

In this case, it is assumed that the one fixed point (LM) would serve as a central station and be used with one slave station to form a single baseline and that the position of a point (EVC) to be located would be on the surface of a solid (sphere, oblate spheroid, etc.) defining the average shape of the Moon. In order to simplify the equations defining the position of the EVC with respect to the fixed location of the LM for this memorandum, it was assumed that the portion of the Moon of interest is a plane surface or flat. It is recognized that this assumption is not realistic if the actual relative position of the EVC is desired. However, it is believed that the calculation of the uncertainty in the determination of the relative position of the EVC on a truly flat Moon given the accuracies of the measurements of the various parameters contained in the equations will provide a reasonable approximation to the similarly calculated uncertainty in the determination of the relative position of the EVC on a more realistic representation of the lunar surface.

With this assumption, the determination of the relative position of the EVC and the LM is reduced to a two-dimensional problem. The LM is defined as the origin of the reference rectangular coordinate system and the positive X-axis is defined in the direction from the LM through the location of the slave tracking station. The coordinates of the LM, the slave tracking station and the EVC in the arbitrary reference coordinate system shown in Figure B-3 will be:

LM: $X = 0,\ Y = 0$

Slave Tracking Station: $X = X_2,\ Y = 0$

EVC, P: $X = X_s,\ Y = Y_s$

The distance from the LM to the EVC is defined as R_1 and the distance from the slave tracking station to the EVC is defined as R_2. The following equations define the position of the EVC with respect to the LM in terms of the location of the slave tracking station with respect to the LM and the distance from the slave tracking station and from the LM to the EVC assuming the portion of the Moon of interest to be a plane surface.

$$X_s = \frac{X_2^2 - R_2^2 + R_1^2}{2X_2}$$

$$Y_s = \left[R_1^{\,2} - X_s^{\,2}\right]^{1/2}$$

Since the uncertainties in the distance measurements and the slave tracking station location were assumed to be mutually independent, the overall position determination uncertainty may be found from the following equations:

$$\Delta X_s = \left\{\left[\left(\frac{+X_2^{\,2}+R_2^{\,2}-R_1^{\,2}}{2X_2^{\,2}}\right)\Delta X_2\right]^2 + \left[\left(\frac{-R_2}{X_2}\right)\Delta R_2\right]^2 + \left[\left(\frac{R_1}{X_2}\right)\Delta R_1\right]^2\right\}^{1/2}$$

$$\Delta Y_s = \left\{\left(\left[\frac{R_1 - X_s\left(\frac{R_1}{X_2}\right)}{\left(R_1^{\,2}-X_s^{\,2}\right)^{1/2}}\right]\Delta R_1\right)^2 + \left(\left[\frac{-X_s}{\left(R_1^{\,2}-X_s^{\,2}\right)^{1/2}}\right]\left[\left(\Delta X\right)^2 - \left(\frac{R_1}{X_2}\,\Delta R_1\right)^2\right]^{1/2}\right)^2\right\}^{1/2}$$

4.3 Two Distance Sums

This case is just a slight variation of the case described in the previous section (4.2) where the sums of two distances are measured instead of each distance separately. The coordinates of the LM, the slave tracking station and the EVC in the arbitrary reference coordinate system shown in Figure B-3 will be:

LM: $X = 0$, $Y = 0$

Slave Tracking Station: $X = X_2$, $Y = 0$

EVC, P: $X = X_s$, $Y = Y_s$

Again it was assumed that the portion of the lunar surface of interest could be approximated by a plane surface.

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The distance from the stationary LM to the EVC plus the distance back to the LM is defined as RS_1 and the distance from the stationary LM to the EVC plus the distance from the crewman to the slave tracking station is defined as RS_2. The following equations define the position of the EVC with respect to the LM in terms of the location of the slave tracking station with respect to the LM and the two measurements of distance sums defined above under the assumption that the lunar surface is a plane.

$$X_s = \frac{X_2^2 - (RS_2)^2 + (RS_1)(RS_2)}{2X_2}$$

$$Y_s = \left[\left(\frac{RS_1}{2}\right)^2 - X_s^2\right]^{1/2}$$

Since the uncertainties in the distance sum measurements and the slave tracking station location were assumed to be mutually independent, the overall position determination uncertainty may be found from the following equations:

$$\Delta X_s = \left\{\left(\left[\frac{X_2^2 + (RS_2)^2 - (RS_1)(RS_2)}{2X_2^2}\right]\Delta X_2\right)^2 + \left(\left[\frac{2RS_2 - RS_1}{-2X_2}\right]\Delta RS_2\right)^2 + \left(\left[\frac{RS_2}{X_2}\right]\Delta RS_1\right)^2\right\}^{1/2}$$

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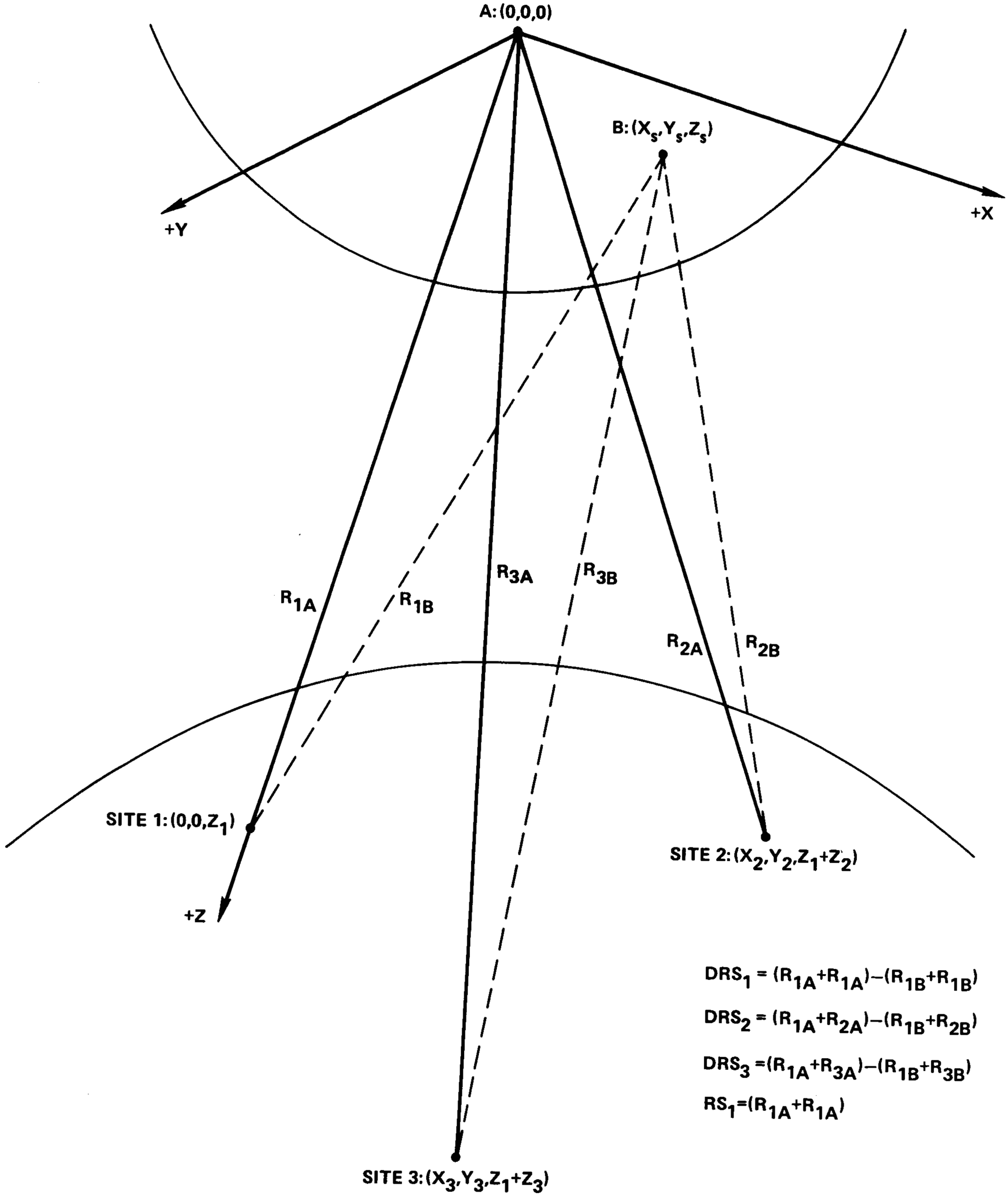
$$\Delta Y_s = \left\{ \left(\frac{1}{2} \left[\left(\frac{RS_1}{2} \right)^2 - X_s^2 \right]^{-1/2} \left[RS_1 - \frac{X_s RS_2}{X_2} \right] \Delta RS_1 \right)^2 + \left(-X_s \left[\left(\frac{RS_1}{2} \right)^2 - X_s^2 \right]^{-1/2} \left[\left(\Delta X_s \right)^2 - \left(\frac{RS_2}{X_2} \Delta RS_1 \right)^2 \right]^{1/2} \right)^2 \right\}^{1/2}$$



A: (0,0,0)
B: (X_s,Y_s,Z_s)
+Y
+X
R_{1A}
R_{1B}
R_{3A}
R_{3B}
R_{2A}
R_{2B}
SITE 1: (0,0,Z_1)
SITE 2: (X_2,Y_2,Z_1+Z_2)
+Z
$DRS_1 = (R_{1A}+R_{1A})-(R_{1B}+R_{1B})$
$DRS_2 = (R_{1A}+R_{2A})-(R_{1B}+R_{2B})$
$DRS_3 = (R_{1A}+R_{3A})-(R_{1B}+R_{3B})$
$RS_1 = (R_{1A}+R_{1A})$
SITE 3: (X_3,Y_3,Z_1+Z_3)

FIGURE B-1

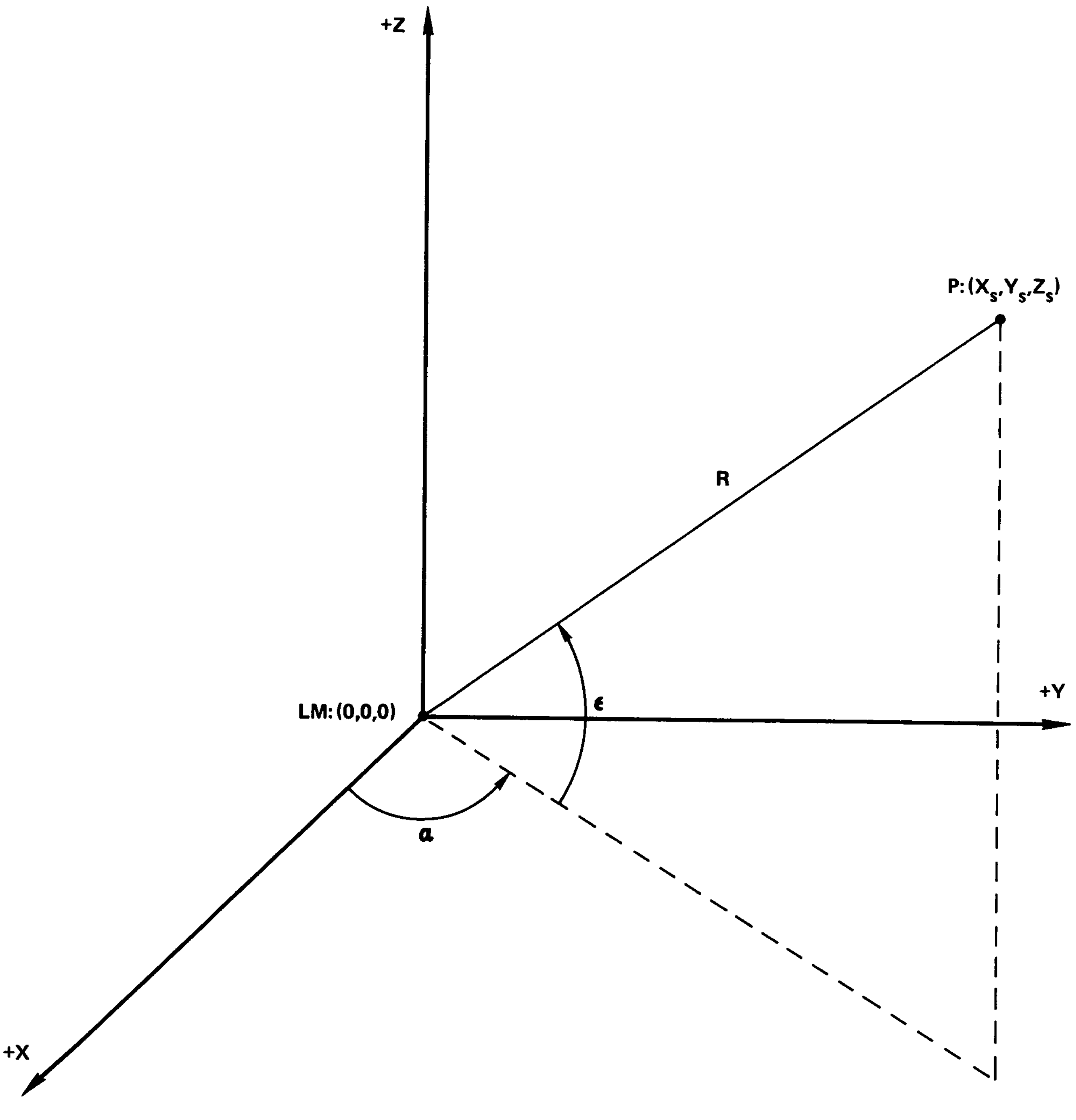
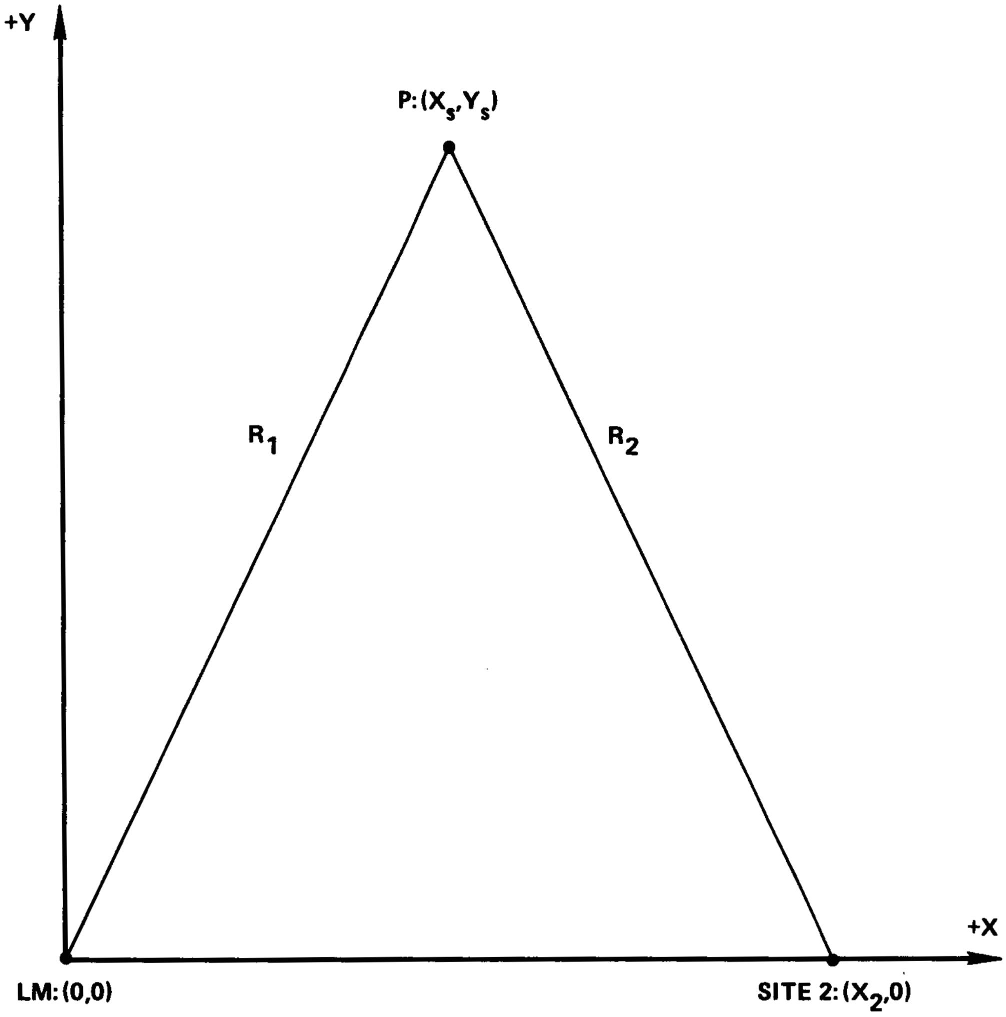


FIGURE B-2



$RS_1 = R_1 + R_1 = 2R_1$

$RS_2 = R_1 + R_2$

FIGURE B-3